AMI #2: Please complete question numbers 32, 34, 48, 60, 80, 102 and 108 from Chapter 5, Section 1 exercises which begin on page 508 of your textbook.

AMI #4: Please complete question numbers 24, 28, 32, and 44 from Chapter 5, Section 2 exercises which begin on page 522 of your textbook.

AMI #2: Work day for AMI #1.

AMI #4: Work day for AMI #3.

Chapter 5 Systems and Matrices

5.1 Systems of Linear Equations

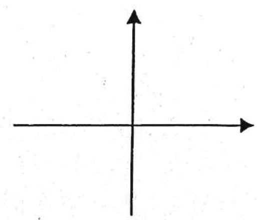
■ **Linear Systems** ■ **Substitution Method** ■ **Elimination Method** ■ **Special Systems**
■ **Applications of Systems of Equations**
■ **Linear Systems with Three Unknowns (Variables)**
■ **Application of Systems to Model Data**

Key Terms: linear equation (first-degree equation) in n unknowns, system of equations, solutions of a system of equations, system of linear equations (linear system), consistent system, independent equations, inconsistent system, dependent equations, equivalent systems, ordered triple

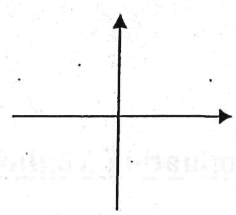
Linear Systems

Possible Graph of a Linear System in Two Unknowns

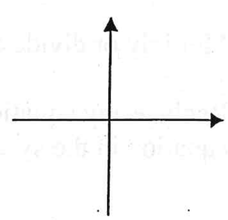
1. **The graphs intersect at exactly one point**, which gives the (single) ordered-pair solution of the system. The **system is** ______________ and the **equations are** ______________.



2. **The graphs are parallel lines,** so there is no solution and the solution set is _____. The **system is** ______________ and the **equations are** ______________.



3. **The graphs are the same line,** and there are an infinite number of solutions. The **system is** ____________ and the **equations are** ______________.



Substitution Method

CLASSROOM EXAMPLE 1 Solving a System (Substitution Method)

Solve the system.

$$\begin{aligned} 4x - 3y &= 14 \\ x - 2y &= 1 \end{aligned}$$

Elimination Method

Transformations of a Linear System

1. Interchange any two equations of the system.
2. Multiply or divide any equation of the system by a nonzero real number.
3. Replace any equation of the system by the sum of that equation and a multiple of another equation in the system.

CLASSROOM EXAMPLE 2 Solving a System (Elimination Method)
Solve the system.

$$2x+3y=-1$$
$$3x-2y=18$$

Special Systems

CLASSROOM EXAMPLE 3 Solving an Inconsistent System
Solve the system.

$$7x+3y=-5$$
$$-14x-6y=-10$$

CLASSROOM EXAMPLE 4 Solving a System with Infinitely Many Solutions

Solve the system.

$$\begin{aligned} -9x+3y &= -24 \\ 3x-\ \ y &= 8 \end{aligned}$$

Application of Systems of Equations

Solving an Applied Problem by Writing a System of Equations

Step 1 ______________ the problem carefully until you understand what is given and what is to be found.

Step 2 ______________________ to represent the unknown values, using diagrams or tables as needed. *Write down* what each variable represents.

Step 3 ______________________________ that relates the unknowns.

Step 4 ______________ the system of equations.

Step 5 ______________________ to the problem. Does it seem reasonable?

Step 6 ______________ the answer in the words of the original problem.

CLASSROOM EXAMPLE 5 Using a Linear System to Solve an Application
In 2015, the average of the median salaries for the position of Environmental Compliance Specialist in Boston, Massachusetts, and Indianapolis, Indiana, was $68,947.50. The median salary in Boston exceeded the median salary in Indianapolis by $7983. Determine the median salary for the Environmental Compliance Specialist position in Boston and in Indianapolis. (*Source:* www.salary.com)

Linear Systems with Three Unknowns (Variables)

Solving a Linear System with Three Unknowns

Step 1 ______________ a variable from any two of the equations.

Step 2 Eliminate the ______________ ______________ from a different pair of equations.

Step 3 ______________ a second variable using the resulting two equations in two variables to obtain an ______________ with just one variable whose value we can now determine.

Step 4 Find the values of the remaining variables by ______________. Write the solution of the system as an ______________ ______________.

CLASSROOM EXAMPLE 6 Solving a System of Three Equations with Three Variables

Solve the system.

$$\begin{aligned} 3x + 4y - 2z &= 14 \\ 2x + y + 2z &= -9 \\ x - y + z &= -9 \end{aligned}$$

CLASSROOM EXAMPLE 7 Solving a System of Two Equations with Three Variables

Solve the system.

$$\begin{aligned} 3x + y - 2z &= -7 \\ 5x + 2y + z &= -6 \end{aligned}$$

Application Systems to Model Data

CLASSROOM EXAMPLE 8 Using Modeling to Find an Equation through Three Points

Find an equation of the parabola $y = ax^2 + bx + c$ that passes through the points (–5, 7), (–1, –2), and (3, 5).

CLASSROOM EXAMPLE 9 Solving an Application Using a System of Three Equations

An animal feed is made from three ingredients: corn, soybeans, and cottonseed. One unit of each ingredient provides units of protein, fat, and fiber as shown in the table. How many units of each ingredient should be used to make a feed that contains 35 units of protein, 38 units of fat, and 28 units of fiber?

	Corn	Soybeans	Cottonseed	Total
Protein	0.25	0.4	0.2	22
Fat	0.4	0.2	0.3	28
Fiber	0.3	0.2	0.1	18

5.2 Matrix Solution of Linear Systems

■ **The Gauss-Jordan Method** ■ **Special Systems**

Key Terms: matrix (matrices), element (of a matrix), augmented matrix, dimension (of a matrix)

The Gauss-Jordan Method

Matrix Row Transformations

For any augmented matrix of a system of linear equations, the following row transformations will result in the matrix of an equivalent system.

1. Interchange any two rows.
2. Multiply or divide the elements of any row by a nonzero real number.
3. Replace any row of the matrix by the sum of the elements of that row and a multiple of the elements of another row.

Using the Gauss-Jordan Method to Put a Matrix into Diagonal Form

Step 1 ____

Step 2 ____

____

Step 3 ____

Step 4 ____

____

Step 5 ____

The Gauss-Jordan method proceeds column by column, from __________ to __________.

CLASSROOM EXAMPLE 1 **Using the Gauss-Jordan Method**

Solve the system.

$$\begin{aligned} 2x + 3y &= 7 \\ 3x - 4y &= -32 \end{aligned}$$

CLASSROOM EXAMPLE 2 Using the Gauss-Jordan Method

Solve the system.

$$\begin{aligned} x + y - 4z &= 10 \\ 2x - 3y + z &= 7 \\ 3x - y - z &= 12 \end{aligned}$$

Special Systems

CLASSROOM EXAMPLE 3 Solving an Inconsistent System
Use the Gauss-Jordan method to solve the system.

$$\begin{aligned} 2x - 3y &= 7 \\ -6x + 9y &= 0 \end{aligned}$$

CLASSROOM EXAMPLE 4 Solving a System with Infinitely Many Solutions
Use the Gauss-Jordan method to solve the system. Write the solution set with z arbitrary.

$$\begin{aligned} 2x + y + z &= 5 \\ 3x + 2y - z &= -8 \end{aligned}$$

Summary of Possible Cases

When matrix methods are used to solve a system of linear equations and the resulting matrix is written in diagonal form (or as close as possible to diagonal form), there are three possible cases.

1. If the number of rows with nonzero elements to the left of the vertical line is equal to the number of variables in the system, then the system has a __________ solution. **See Examples 1 and 2.**
2. If one of the rows has the form $[0 \ 0 \ \cdots \ 0 \mid a]$ with $a \neq 0$, then the system has ______ solution. **See Example 3.**
3. If there are fewer rows in the matrix containing nonzero elements than the number of variables, then the system has either ______ solution or __________ __________ solutions. If there are __________ __________ solutions, give the solutions in terms of one or more arbitrary variables. **See Example 4.**