Week One

Algebra II

Desoto County Schools

Factoring Methods for $ax^2 + bx + c$

ANY METHOD: Before factoring, factor out the GCF
ANY METHOD: After factoring, check by multiplying to verify original polynomial

Guess and Check

1. Factor out GCF.
2. Draw parentheses.
3. Find factors of a; find factors of c.
4. Try different pairings of factors until a pair works.

Factor $10x^2 + 21x + 8$

Factors of 10: 1·10 and 2·5; Factors of 8: 1·8, 2·4

(1x + 1)(10x + 8) No

(1x + 8)(10x +1) No

(1x + 2)(10x +4) No

(1x + 4)(10x + 2) No

(5x + 8)(2x+1) Yes

Grouping

1. Factor out GCF.
2. Multiply ac.
3. Find two factors of ac that add or subtract to b.
4. Split bx term into sum of those two numbers.
5. Group first two terms and last two terms (reverse distribute).
6. Factor the common polynomial.

Factor $10x^2 + 21x + 8$

10·8 = 80. Factors of 80 that add to 21: 16 and 5

$10x^2 + 16x + 5x + 8$

2x(5x + 8) + (5x + 8)

(5x + 8)(2x + 1)

Box Method

1. Factor out GCF.
2. Draw a 2x2 box.
3. Put first term (ax^2) in top left, last term (c) in bottom right
4. Multiply ac; find factors of ac that add to middle term b. Put these terms in top right and bottom left boxes.
5. Factor the GCF from each row and column.
6. These values make up the factors!

Factor $10x^2 + 21x + 8$

10·8 = 80. Factors of 80 that add to 21: 16 & 5

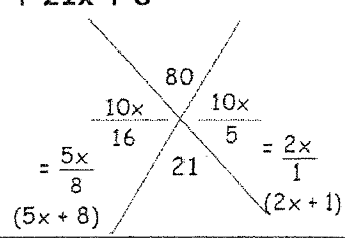
	5x	8
2x	$10x^2$	16x
1	5x	8

Factors: (5x + 8)(2x + 1)

Diamond Method

1. Factor out GCF.
2. Draw a big X. Multiply ac and put in top of x; put b in bottom of x.
3. Find two factors of ac that add to b.
4. Put those factors on the left and right of the "X," but make as denominators of fractions.
5. Make leading coefficient multiplied by variable as the numerator of the fraction.
6. Reduce fractions if possible. These are your factors!

Factor $10x^2 + 21x + 8$



Slide and Divide

1. Factor out GCF.
2. Multiply ac and rewrite the trinomial with a leading coefficient of 1 and the third term as the product of ac ("slide").
3. Factor using strategies when leading coefficient is 1 (type III factoring).
4. Divide each numerical term by the original leading coefficient, and reduce to simplest form ("divide").
5. Multiply the terms in each set of parentheses by the LCD of the two terms.

Factor $10x^2 + 21x + 8$

ac = 80; rewrite: $x^2 + 21x + 80$

Factor: (x + 16)(x + 5)

Divide by 10: $(x + \frac{16}{10})(x + \frac{5}{10})$

Reduce: $(x + \frac{8}{5})(x + \frac{1}{2})$

Multiply by LCD: (5x + 8)(2x + 1)

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Factoring $x^2 + bx + c$

EXAMPLE 2

Factor $x^2 - 7x + 12$.

SOLUTION

You want $x^2 - 7x + 12 = (x + m)(x + n)$ where $mn = 12$ and $m + n = -7$.

Factors of 12	1, 12	2, 6	3, 4	−1, −12	−2, −6	−3, −4
Sum of factors $(m + n)$	13	8	7	−13	−8	−7

The table shows that the values of m and n you want are $m = -3$ and $n = -4$. So, $x^2 - 7x + 12 = (x - 3)(x - 4)$.

Factor.

4. $x^2 - 12x + 35$ **5.** $x^2 - 9x + 18$ **6.** $x^2 - 10x + 25$

In Example 3, the same method is used as in Example 2, but notice that both b and c are negative.

EXAMPLE 3

Factor $x^2 - 3x - 18$.

SOLUTION

You want $x^2 - 3x - 18 = (x + m)(x + n)$ where $mn = -18$ and $m + n = -3$.

Factors of -18	−1, 18	1, −18	−2, 9	2, −9	−3, 6	3, −6
Sum of factors $(m + n)$	17	−17	7	−7	3	−3

The table shows that the values of m and n you want are $m = 3$ and $n = -6$. So, $x^2 - 3x - 18 = (x + 3)(x - 6)$.

Factor.

7. $x^2 - 6x - 7$ **8.** $x^2 - 5x - 24$ **9.** $x^2 - 3x - 54$

In Example 4, b is positive and c is negative.

EXAMPLE 4

Factor $x^2 + x - 20$.

SOLUTION

You want $x^2 + x - 20 = (x + m)(x + n)$ where $mn = -20$ and $m + n = 1$.

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LESSON 5.2 CONTINUED

Topic 5

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Factoring $x^2 + bx + c$

Factors of -20	$-1, 20$	$1, -20$	$-2, 10$	$2, -10$	$-4, 5$	$4, -5$
Sum of factors $(m + n)$	19	-19	8	-8	1	-1

The table shows that the values of m and n you want are $m = -4$ and $n = 5$. So, $x^2 + x - 20 = (x - 4)(x + 5)$.

Factor.

10. $x^2 + 4x - 12$

11. $x^2 + 4x - 45$

12. $x^2 + x - 56$

Mixed Review

State the inverse.

13. Add -16.

14. Subtract 35.

15. Divide by $\frac{2}{3}$.

Solve the inequality.

16. $x + 9 < 16$

17. $-6x \geq 36$

18. $\frac{x}{7} > -5$

19. ***Find the Numbers*** The sum of two numbers is 34. The second number is two more than the first. Find the two numbers.

Topic 5

LESSON 5.2

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Practice

For use with Lesson 5.2: Factoring $x^2 + bx + c$

Match the trinomial with the correct factorization.

1. $x^2 + x - 12$
2. $x^2 + 7x + 12$
3. $x^2 - 7x + 12$
4. $x^2 - x - 12$

A. $(x + 4)(x + 3)$
B. $(x - 4)(x - 3)$
C. $(x + 4)(x - 3)$
D. $(x - 4)(x + 3)$

Choose the correct factorization. If neither is correct, find the correct factorization.

5. $x^2 + 14x + 48$
   A. $(x + 6)(x + 8)$
   B. $(x + 4)(x + 12)$

6. $x^2 - 3x - 10$
   A. $(x - 2)(x + 5)$
   B. $(x - 5)(x + 2)$

7. $x^2 + 8x - 33$
   A. $(x + 3)(x - 11)$
   B. $(x - 3)(x - 11)$

Factor the trinomial.

8. $x^2 + 8x - 9$
9. $x^2 - 10x + 21$
10. $x^2 + 5x - 24$
11. $x^2 + 13x + 36$
12. $x^2 - 3x - 18$
13. $x^2 + 14x + 40$
14. $x^2 - x - 56$
15. $x^2 - 7x - 30$
16. $x^2 + 12x + 32$
17. $x^2 + 3x - 54$
18. $x^2 - 2x - 15$
19. $x^2 - 20x + 100$
20. $x^2 + 2x - 63$
21. $x^2 - 10x - 24$
22. $x^2 + 16x + 39$
23. $x^2 + 6x - 55$
24. $x^2 - 9x - 70$
25. $x^2 - 22x + 40$

LESSON 5.3

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Factoring $ax^2 + bx + c$

GOAL **Factor quadratic expressions of the form $ax^2 + bx + c$.**

Understanding the Main Ideas

In this lesson, you will learn how to factor quadratic polynomials whose leading coefficient is not 1. To do this, find the factors of $a(m$ and $n)$ and the factors of $c(p$ and $q)$ so that the sum of the outer and inner products (mq and pn) is b.

$$c = pq$$

$$ax^2 + bx + c = (mx + p)(nx + q) \qquad b = mq + pn$$

$$a = mn$$

$$20 = 5 \cdot 4$$

Example: $6x^2 + 22x + 20 = (3x + 5)(2x + 4) \qquad 22 = (3 \cdot 4) + (5 \cdot 2)$

$$6 = 3 \cdot 2$$

Once you determine the factors of a and c, it is necessary to test them to see which produces the correct factorization. In Example 1, there is only one pair of factors for a and c.

EXAMPLE 1

Factor $2x^2 + 5x + 3$.

SOLUTION

Test the possible factors of a(1 and 2) and c(1 and 3).

Try $a = 1 \cdot 2$ and $c = 3 \cdot 1$.

$(1x + 3)(2x + 1) = 2x^2 + 7x + 3$ Not correct

Now try $a = 1 \cdot 2$ and $c = 1 \cdot 3$.

$(1x + 1)(2x + 3) = 2x^2 + 5x + 3$ Correct

The correct factorization of $2x^2 + 5x + 3$ is $(x + 1)(2x + 3)$.

Factor the expression.

1. $3x^2 + 2x - 1$ **2.** $2x^2 + 11x + 5$ **3.** $5x^2 + 8x + 3$

For more complicated expressions like that in Example 2 and Example 3 where there are several pairs of factors for a and c, it is convenient to set up a table when testing the factors.

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Factoring $ax^2 + bx + c$

EXAMPLE 2

Factor $2x^2 - 3x - 5$.

SOLUTION

Factors of a and c	Product	Correct?
$a = 1 \cdot 2$ and $c = (-1)(5)$	$(x - 1)(2x + 5) = 2x^2 + 3x - 5$	No
$a = 1 \cdot 2$ and $c = (5)(-1)$	$(x + 5)(2x - 1) = 2x^2 + 9x - 5$	No
$a = 1 \cdot 2$ and $c = (1)(-5)$	$(x + 1)(2x - 5) = 2x^2 - 3x - 5$	Yes
$a = 1 \cdot 2$ and $c = (-5)(1)$	$(x - 5)(2x + 1) = 2x^2 - 9x - 5$	No

The correct factorization of $2x^2 - 3x - 5$ is $(x + 1)(2x - 5)$.

Factor the expression.

4. $3x^2 - 4x - 7$
5. $5x^2 - 14x - 3$
6. $7x^2 + 13x - 2$

EXAMPLE 3

Factor $8x^2 - 26x + 15$.

SOLUTION

Both factors of c must be negative, because b is negative and c is positive. Test the possible factors of a and c.

Factors of a and c	Product	Correct?
$a = 1 \cdot 8$ and $c = (-15)(-1)$	$(x - 15)(8x - 1) = 8x^2 - 121x + 15$	No
$a = 1 \cdot 8$ and $c = (-1)(-15)$	$(x - 1)(8x - 15) = 8x^2 - 23x + 15$	No
$a = 1 \cdot 8$ and $c = (-3)(-5)$	$(x - 3)(8x - 5) = 8x^2 - 29x + 15$	No
$a = 1 \cdot 8$ and $c = (-5)(-3)$	$(x - 5)(8x - 3) = 8x^2 - 43x + 15$	No
$a = 2 \cdot 4$ and $c = (-15)(-1)$	$(2x - 15)(4x - 1) = 8x^2 - 62x + 15$	No
$a = 2 \cdot 4$ and $c = (-1)(-15)$	$(2x - 1)(4x - 15) = 8x^2 - 34x + 15$	No
$a = 2 \cdot 4$ and $c = (-3)(-5)$	$(2x - 3)(4x - 5) = 8x^2 - 22x + 15$	No
$a = 2 \cdot 4$ and $c = (-5)(-3)$	$(2x - 5)(4x - 3) = 8x^2 - 26x + 15$	Yes

The correct factorization of $8x^2 - 26x + 15$ is $(4x - 3)(2x - 5)$.

Factor the expression.

7. $4x^2 - 12x + 9$
8. $8x^2 - 26x + 21$
9. $9x^2 + 18x - 16$

If a, b, and c have a common factor, factor out the common factor before testing the possible factors of a and c, as shown in Example 4.

(continued)

LESSON 5.3

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Topic 5

Practice

For use with Lesson 5.3: Factoring $ax^2 + bx + c$

Match the trinomial with the correct factorization.

1. $3x^2 + 14x + 8$	A. $(3x - 2)(x + 4)$
2. $3x^2 - 23x - 8$	B. $(3x + 1)(x - 8)$
3. $3x^2 + 23x - 8$	C. $(3x + 2)(x + 4)$
4. $3x^2 + 10x - 8$	D. $(3x - 1)(x + 8)$

Choose the correct factorization. If neither is correct, find the correct factorization.

5. $3x^2 + 7x - 6$
   A. $(3x - 1)(x + 6)$
   B. $(3x - 2)(x + 3)$

6. $6x^2 - 7x - 3$
   A. $(3x - 1)(2x - 3)$
   B. $(6x - 1)(x + 3)$

7. $4x^2 - 21x + 5$
   A. $(4x - 1)(x - 5)$
   B. $(2x - 1)(2x - 5)$

Factor the trinomial.

8. $2x^2 + 9x + 7$
9. $3x^2 - 8x - 16$
10. $4x^2 - 16x + 15$
11. $5x^2 + 12x - 9$
12. $4x^2 + 11x + 6$
13. $6x^2 - 23x + 20$
14. $6x^2 - 3x - 3$
15. $8x^2 + 42x - 36$
16. $7x^2 + 33x - 10$
17. $4x^2 - 10x - 14$
18. $4x^2 + 21x + 35$
19. $9x^2 - 12x - 12$
20. $5x^2 + 41x - 36$
21. $6x^2 + 3x - 30$
22. $7x^2 - 59x - 36$
23. $4x^2 + 37x + 40$
24. $10x^2 - 27x + 18$
25. $8x^2 + 26x + 21$

Factoring

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Factor.

1. $6x^2 - 25x - 9$ (2x − 9)(3x + 1)

2. $15x^2 - 38x + 7$

3. $21x^2 + x - 2$

4. $15x^2 + 8x + 1$

5. $25x^2 + 55x + 18$

6. $25x^2 - 30x - 16$

7. $6x^2 - 11x - 10$

8. $10x^2 - 3x - 27$

9. $24x^2 - 2x - 15$

10. $42x^2 + 5x - 25$

11. $21x^2 + 11x - 6$

12. $4x^2 + 20x + 9$

13. $12x^2 + 20x + 7$

14. $35x^2 - 58x - 9$

15. $28x^2 - 37x + 12$

16. $20x^2 - 9x - 18$

17. $25x^2 - 10x - 63$

18. $49x^2 + 21x - 4$

19. $21x^2 + 19x - 12$

20. $25x^2 - 35x + 12$

21. $16x^2 + 32x + 15$

22. $6x^2 + 11x + 3$

23. $10x^2 - 43x + 28$

24. $35x^2 - 6x - 8$

25. $20x^2 - 23x + 6$

26. $14x^2 - 45x - 14$

27. $4x^2 - 4x - 3$

28. $28x^2 + 15x + 2$

29. $12x^2 + x - 63$

30. $49x^2 - 42x - 16$

31. $18x^2 + 15x + 2$

32. $24x^2 + 26x - 63$

33. $30x^2 + 73x + 7$

34. $50x^2 - 45x - 18$

LESSON 5.4

Topic 5

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Factoring Special Cases

GOAL **Use special product patterns to factor quadratic polynomials.**

Terms to Know	*Example/Illustration*
Prime factor a factor that cannot be factored using integer coefficients	$x^2 + 5x + 6 = \underbrace{(x + 3)(x + 2)}_{\text{Prime factors}}$
Factoring a polynomial completely to write a polynomial as the product of: • monomial factors • prime factors with at least two terms	$2x^2 - 8 = 2(x^2 - 4)$ $= 2(x - 2)(x + 2)$

Understanding the Main Ideas

When factoring quadratic polynomials, it is helpful to be able to recognize special product patterns.

Factoring Special Products

Difference of Two Squares Pattern

$a^2 - b^2 = (a + b)(a - b)$

Example:

$9x^2 - 4 = (3x + 2)(3x - 2)$

Perfect Square Trinomial Pattern

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example:

$x^2 + 10x + 25 = (x + 5)^2$

$x^2 - 14x + 49 = (x - 7)^2$

EXAMPLE 1

Factor the difference of two squares.

a. $x^2 - 9$ **b.** $4x^2 - 36$ **c.** $48 - 75x^2$

SOLUTION

a. $x^2 - 9 = x^2 - 3^2$ Write as $a^2 - b^2$.

$= (x + 3)(x - 3)$ Factor using pattern.

b. $4x^2 - 36 = (2x)^2 - 6^2$ Write as $a^2 - b^2$.

$= (2x + 6)(2x - 6)$ Factor using pattern.

c. $48 - 75x^2 = 3(16 - 25x^2)$ Factor out common factor.

$= 3[4^2 - (5x)^2]$ Write as $a^2 - b^2$.

$= 3(4 + 5x)(4 - 5x)$ Factor using pattern.

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