

Limits

A function $f(x)$ has a limit as x approaches c **if and only** if the right-hand and left-hand limits at c exist and are equal.

$$\lim_{x \rightarrow c} f(x) = L \iff \lim_{x \rightarrow c^-} f(x) = L \text{ and } \lim_{x \rightarrow c^+} f(x) = L$$

Consider

$$\lim_{x \rightarrow 2} (x^2 - 5x)$$

Read this as either

“The limit as x goes to two, of x squared minus five x .”

or

“The limit of x squared minus five x , as x goes to two.”

For the following limit theorems, assume:

$$\lim_{x \rightarrow a} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = M$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = \left[\lim_{x \rightarrow a} f(x) \right] \cdot \left[\lim_{x \rightarrow a} g(x) \right] = L \cdot M$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}$$

$$\lim_{x \rightarrow a} (k \cdot f(x)) = k \lim_{x \rightarrow a} f(x) = k L$$

(where k is a constant)

To find $\lim_{x \rightarrow a} f(x)$

- If $f(x)$ is a polynomial, simply evaluate $f(a)$.
- If $f(x)$ is not a polynomial (such as a rational expression), try to evaluate $f(a)$ unless it gives some indeterminate form such as:
 - Division by zero
 - Undefined
 - ∞/∞ , $0/0$, etc.

In these cases try to algebraically eliminate the difficulty before substituting in the a value.

If it is not possible to eliminate the indeterminate form, the limit is either

∞ , $-\infty$, or DNE

Example 1) find $\lim_{x \rightarrow -2} x^2 - 4x + 1$

You can do this by plugging in.

Example 2) find $\lim_{x \rightarrow -2} \frac{2x-6}{x-2}$

You can also do this by plugging in.

Example 3) find $\lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{x^2 - 4}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

Example 4: Find

$$\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 4}$$

Example 5: Find $\lim_{x \rightarrow 4} \frac{5x - 20}{x^2 - 16}$

Example 6: Find $\lim_{x \rightarrow 3} \left(\frac{x^2 + 2x - 15}{x - 3} \right)$

Example 7: Find

$$\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 4x + 3}$$

Homework: Complete #1-21 on the Handout