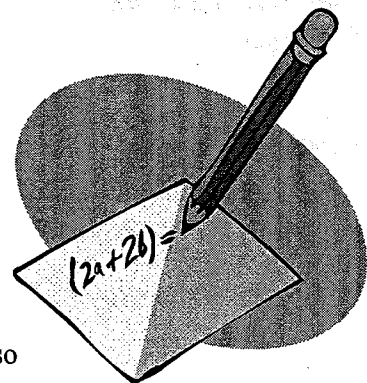


# Properties of Real Numbers

## Section 2.1 Basic Properties



There are certain properties of mathematics that allow you to manipulate and simplify expressions and equations. You may not know their names, but you should know how to use them. It's a little like recognizing the faces of people you've met without being able to call them by name. It's time to put the "names with the faces," so to speak, and make sure you know what you can do with these basic properties.

### Commutative Property

The commutative property simply says that numbers can be added in any order or multiplied in any order without changing the sum or product of the numbers. If you use symbols instead of numbers, you get the Universal Formula for Commutative Property.

For all numbers $a$ & $b$	
Addition	Multiplication
$a + b = b + a$	$a \cdot b = b \cdot a$

### Associative Property

The associative property allows you to regroup the addends in addition and the factors in multiplication without changing the sum or product.

For all numbers $a$ , $b$ and $c$	
Addition	
$(a + b) + c = a + (b + c)$	
Multiplication	
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	

### Addition & Subtraction Property of Zero

The addition and subtraction property of zero means that you can add zero to any number or subtract zero from any number without changing the number.

For any number $a$	
Addition	Subtraction
$a + 0 = a$	$a - 0 = a$

### Multiplication & Division Property of One

The multiplication and division property of one allows you to multiply or divide any number by one without changing the number.

For any number $a$	
Multiplication	Division
$a \cdot 1 = a$	$a \div 1 = a$

### Multiplication & Division Property of Zero

You can multiply zero by any number or divide zero by any number, and the answer will be zero. Notice that this property doesn't say that you could divide any number by zero. Division by zero is not permitted. The solution to any number divided by zero is *undefined*.

For all real numbers $a$
$0 \cdot a = 0$

For all real numbers $a$ with $a \neq 0$
$0 \div a = 0$ or $\frac{0}{a} = 0$

<b>CAUTION!</b>
$\frac{a}{0} = \text{undefined}$

## Section 2.1, continued

### Basic Properties

#### Distributive Property

The distributive property actually involves two operations: multiplication with addition or subtraction. It allows you to simplify expressions that have a number times a sum or difference in parentheses. The distributive property allows you to “distribute” the multiplication to each of the terms in the parentheses. Notice that it eliminates the parentheses. The distributive property is very important when simplifying algebraic expressions and equations.

For all real numbers $a$ , $b$ and $c$
Addition
$a(b + c) = a \cdot b + a \cdot c$
Subtraction
$a(b - c) = a \cdot b - a \cdot c$

**Example 1:** Simplify the expression  $2(x - 4)$ .

**Step 1:** Using the distributive property, the “2” is distributed by multiplying it to both terms inside the parentheses.  $2(x - 4) \rightarrow 2 \cdot x - 2 \cdot 4$

**Step 2:** Simplify by multiplying.  $2x - 8$

**Example 2:** Simplify the expression  $3.4(2.5 + n)$ .

Decimal numbers are treated exactly the same. If you don’t want to do the multiplication by hand or in your head, use a calculator.

**Step 1:** Use the distributive property and multiply both terms inside the parentheses by 3.4.  $3.4(2.5 + n) \rightarrow 3.4 \cdot 2.5 + 3.4 \cdot n$

**Step 2:** Simplify by multiplying.  $8.5 + 3.4n$

**Example 3:** Simplify the expression  $-(a + 7)$ .

**Step 1:** A minus sign can be distributed just like a number. A minus sign like the one shown here can be thought of as a type of shorthand for  $-1$ , but the “1” is not written.  $-(a + 7) \rightarrow -1(a + 7)$

**Step 2:** Use the distributive property to distribute the  $-1$  (or just the  $(-)$  sign if you want to think of it that way.)  $-1 \cdot a + (-1) \cdot 7$

**Step 3:** Simplify by multiplying. When a variable has a coefficient of “1,” the “1” is often not written, even when it is a negative “1.” Therefore,  $-1a$  can be written as  $-a$ .  $-a + (-7)$

Also, you may remember from Section 1.3 that adding a negative number is the same as subtracting a positive one, so adding negative 7 is the same as subtracting positive 7.  $-a - 7$

## Section 2.1, continued

### Basic Properties

**Example 4:** Simplify the expression  $-8(y - 9)$ .

When you have one or more negative numbers and/or subtraction, be careful with the signs!

- Step 1:** To remove the parentheses,  $-8$  is multiplied to each inside term.  $-8(y - 9) \rightarrow -8 \cdot y - (-8)(9)$
- Step 2:** Do the multiplication.  $-8y - (-72)$
- Step 3:** Remember that subtracting a negative number is the same as adding a positive one, so simplify the signs.  $-8y + 72$

### Practice 1

Review the basic properties given in this sub-section. Based on these properties, determine if the following mathematical statements are equal or not. If both sides of the equation are equal, write an *E* in the blank. If the sides are not equal, write an *N* in the blank.

**Examples:**   E    $5 \cdot x = x \cdot 5$         N    $5 \div x = x \div 5$

- |                                    |  |   |
|------------------------------------|--|---|
| <u>      </u> 1. $x + 2 = 2 + x$   | <u>      </u> 2. $(x + 4) + 5 = x + (4 + 5)$                 | <u>      </u> 3. $1 \cdot 2x = 2x$            |
| <u>      </u> 4. $0 \cdot 4x = 4x$ | <u>      </u> 5. $2 \cdot (3 \cdot x) = (2 \cdot 3) \cdot x$ | <u>      </u> 6. $x + 0 = 0$ (if $x \neq 0$ ) |
| <u>      </u> 7. $0 \div 4 = 0$    | <u>      </u> 8. $(7 \cdot x) \div 4 = 7 \cdot (x \div 4)$   | <u>      </u> 9. $x - 0 = x$                  |
| <u>      </u> 10. $x \div 1 = x$   | <u>      </u> 11. $4 - x = x - 4$                            | <u>      </u> 12. $2x \div 1 = 1$             |

### Practice 2

Use the distributive property to simplify the following problems. Write your final answer in each blank.

1. $6(x + 3)$ <u>                  </u>	2. $-4(y + 7)$ <u>                  </u>	3. $2(x - 5)$ <u>                  </u>	4. $-5(a - 1)$ <u>                  </u>
5. $1.1(x + 3)$ <u>                  </u>	6. $1.5(2.2 - x)$ <u>                  </u>	7. $-5.7(n - 4)$ <u>                  </u>	8. $-3(2.9 + b)$ <u>                  </u>

**Section 2.1, continued**  
**Basic Properties**

**Practice 3**

Use the basic properties of mathematics to answer the following questions. Darken the circle that represents the correct answer.

1. Which of the following is equivalent to the expression  $x(y - z)$  for real numbers  $x$ ,  $y$ , and  $z$ ?

- A.  $xy - z$
- B.  $xy - xz$
- C.  $x + y - x + z$
- D.  $x - y + x - z$

(A) (B) (C) (D)

5. If  $x = 0$ , which of these expression is equal to 0?

- A.  $(x + 1) \cdot 3$
- B.  $x + (2 - 1 \cdot 3)$
- C.  $x \cdot (3 + 4 - 1)$
- D.  $(x - 4) \cdot (4 + 5)$

(A) (B) (C) (D)

2. Which of the following is equivalent to the expression  $x + 3$ ?

- A.  $x \div 3$
- B.  $3x$
- C.  $x - 3$
- D.  $3 + x$

(A) (B) (C) (D)

6. Which of the following is equivalent to the equation  $2(a + 5) = 3(a - 1)$ ?

- A.  $2a + 5 = 3a - 1$
- B.  $2a + 10 = 3a - 3$
- C.  $2a + 5 = 3a$
- D.  $2a + 10 = 3a$

(A) (B) (C) (D)

3. If  $y = 1$ , which of the following expressions is equivalent to  $x + 2$ ?

- A.  $y + x + 2$
- B.  $y(x + 2)$
- C.  $y - x + 2$
- D.  $2 \cdot (x + y)$

(A) (B) (C) (D)

7. For all real numbers  $a$ ,  $b$ , and  $c$ , which of the following is equivalent to the expression  $a \cdot (b + c)$ ?

- A.  $b \cdot (a + c)$
- B.  $ab + c$
- C.  $(c + b) \cdot a$
- D.  $bc + ac$

(A) (B) (C) (D)

4. If  $a = 0$ , which of the following expressions is equivalent to 7?

- A.  $7a$
- B.  $7 \div a$
- C.  $a \div 7$
- D.  $7 + a$

(A) (B) (C) (D)

8. For all real numbers  $x$ ,  $y$ , and  $z$ , which of the following expression is equivalent to  $x + y$  if  $z = 0$ ?

- A.  $x + y - z$
- B.  $z \cdot (x - y)$
- C.  $xz + y$
- D.  $x \cdot (y + z)$

(A) (B) (C) (D)

# Properties of Real Numbers

## Section 2.2 Introduction to Order of Operations



When expressions have more than one term combined by a math operation, it's important to know which operation should be done first. The rules that determine what sequence to follow in simplifying expressions is called **order of operations**. Let's take a look.

$$2 + 4 \cdot 5 = ?$$

$$6 \cdot 5 \stackrel{?}{=} 30 \quad \times$$

$$2 + 20 \stackrel{?}{=} 22 \quad \checkmark$$

If you add first, you get  $6 \cdot 5$ , or 30. If you multiply first, you get  $20 + 2$ , or 22. Which is right? That's why order of operations is so important. It tells you the correct sequence for simplifying expressions with mixed operations. The correct answer is 22. That means multiplication is to be done before addition.

**The rules for simple arithmetic operations is to work the following order :**

1. Exponents
2. Multiplication or Division
3. Addition or Subtraction

We'll get to exponents in a minute. First look at multiplication, division, addition, and subtraction. As you can see, multiplication and division are on the same level, and addition and subtraction are on the same level. As long as you are deciding between operations on different levels, the order is clear. But what if you have multiple operations on the same level like the following?

**Example 1:** Simplify  $60 \div 6 \cdot 5 \div 2 - 3 + 10 - 7$

The **Left Hand Rule** becomes very important in solving this problem. It says that when you have multiple operations on the same level, you simplify them from left to right.

**Step 1:** In this case, you would begin on the far left with the first division operation  $60 \div 6$  which is 10.

**Step 2:** Multiply  $10 \cdot 5$  to get 50.

**Step 3:** Divide 50 by 2 to get 25.

**Step 4:** Next, move to the addition and subtraction level. Left to right would mean that you subtract 3 from 25 to get 22.

**Step 5:** Add 22 and 10 for a total of 32.

**Step 6:** The last step would be to subtract 7 from 32 to get 25.

$$\boxed{60 \div 6} \cdot 5 \div 2 - 3 + 10 - 7 =$$

$$\boxed{10 \cdot 5} \div 2 - 3 + 10 - 7 =$$

$$\boxed{50 \div 2} - 3 + 10 - 7 =$$

$$\boxed{25 - 3} + 10 - 7 =$$

$$\boxed{22 + 10} - 7 =$$

$$32 - 7 = 25$$

In each step, the Left Hand Rule was used to determine the order when the operations were on the same level. But what if you wanted a particular order for the math to be done that did not follow the rules? Then you would have to group the operations using grouping symbols.

**Section 2.2, continued**  
**Introduction to**  
**Order of Operations**

Addition, subtraction, multiplication, and division are the main arithmetic operations, but they aren't the only ones. What about exponents? Exponents should be simplified before multiplying or dividing.

**Example 2:** Simplify the expression  $4 - 2^2 \times 3$ .

- |  |   |
|--|---|
| <b>Step 1:</b> First, perform the operation on the term with the exponent. $2^2$ is 4. | $4 - 2^2 \times 3 \rightarrow 4 - 4 \times 3$ |
| <b>Step 2:</b> Next do the multiplication. $4 \times 3$ is 12.                         | $4 - 12$                                      |
| <b>Step 3:</b> Finally, do the subtraction.  | $-8$  |

Now look at a problem that requires substitution. Use the correct order of operations to simplify.

**Example 3:** What is the value of the expression below when  $x = -4$ ?

$$x^2 + 3x - 9$$

- |   |                                      |
|---|--------------------------------------|
| <b>Step 1:</b> To simplify, you must first substitute $-4$ for each $x$ in the expression.  | $(-4)^2 + 3(-4) - 9$                 |
| <b>Step 2:</b> Exponents are done first. Remember that when you raise a negative number to an even power, you get a positive value. | $16 + 3(-4) - 9$                     |
| <b>Step 3:</b> Next do the other multiplication. Be careful with signs!   | $16 + (-12) - 9$ or<br>$16 - 12 - 9$ |
| <b>Step 4:</b> Finally, do the subtraction from left to right.  | $4 - 9 = -5$                         |

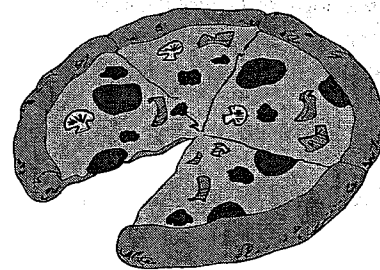
**Practice**

Simplify the following expressions using the correct order of operations. Show your work, and write your final answer in the blank.

_____ 1. $3 \times 4 \div 2 - 3 + 4$	_____ 2. $7 - (-4)^2 \times 2$	_____ 3. $6 \times 3 + (-3)^3$
_____ 4. $y^2 - 2y + 3$ when $y = -2$	_____ 5. $x^3 + 7x^2 - x$ when $x = -1$	_____ 6. $15 - 4a - a^3$ when $a = 2$

# Properties of Real Numbers

## Section 2.3 Order of Operations with Grouping Symbols



Let's take order of operations a step further to consider different types of grouping symbols. The main grouping symbols are shown below.

$( )$   
Parentheses

$[ ]$   
Brackets

$\{ \}$   
Braces

$||$   
Absolute Value

$\frac{\quad}{\quad}$   
Fraction Bar

$\sqrt{\quad}$   
Radical Symbol

Anytime you see one of these grouping symbols, it must be simplified first. Put everything together, and you get the complete order of operations as shown below.

1. Grouping Symbols
2. Exponents
3. Multiplication & Division
4. Addition & Subtraction

To help you remember the order, make a sentence with the first letters of the list. A common one is given below:

**Greatly Excuse My Dear Aunt Sally**

$\swarrow$  Grouping Symbols     $\swarrow$  Exponents     $\swarrow$  Multiplication & Division     $\swarrow$  Addition & Subtraction

### Parentheses, Braces, and Brackets

Parentheses, braces, and brackets are familiar grouping symbols. When you see them, you know that the contents inside go together and must be simplified before you do the other operations.

When would you want to do use parentheses? You may use them more often than you realize. Look at the example below.

**Example 1:** Let's say you and three friends are having pizza. You bought two mediums and a large. The mediums have 6 slices each, and the large has eight slices. To give each person an equal share, how many slices should each person get?

In this problem, you need to add together the total number of slices first and then divide by the total number of people. If you write out the problem without using grouping symbols, the rules for order of operations would have you divide first, and you would not get the correct answer. So, you need to use parentheses (the most commonly used grouping symbol) to show that you add first and then divide.

$$\text{pizza } \frac{6 + 6 + 8}{1 + 3} \text{ people}$$

$$(6 + 6 + 8) \div (1 + 3)$$

$$20 \div 4 = 5$$

Absolute value, fraction bars, and radical symbols are also considered grouping symbols that you need to know.

### Absolute Value Bars

Absolute value has to do with how far a number is from zero on a number line. To determine the absolute value, you must first perform any operations inside the bars, and then see how many units away from zero the number falls.

## Section 2.3, continued

### Order of Operations with Grouping Symbols

**Example 2:** Simplify the expression  $|3 - 6| - 2$ .

Since absolute value bars are considered grouping symbols, you must simplify any operations inside the bars first.

$$|3 - 6| - 2$$

**Step 1:** Subtract 6 from 3 first since this operation is inside the absolute value bars. The result is the absolute value of  $-3$ .

$$|-3| - 2$$

**Step 2:** Next, determine the absolute value of  $-3$ , which is 3. To make it simple, absolute value is the number with no positive or negative sign. It's just the distance from zero. Here that would be 3 units.

$$3 - 2$$

**Step 3:** Finally, perform any other operations. The solution would be  $3 - 2$  or 1. You will see absolute value in more detail later. For now, all you need to do is recognize that the absolute value symbol is a grouping symbol.

$$1$$

### Fraction Bars

Fraction bars show that one number is to be divided by another. You can also have terms separated by a fraction bar. When terms are separated by a fraction bar, the fraction bar becomes a grouping symbol that says do the math in the numerator and the denominator before you try to divide.

**Example 3:** Simplify the expression  $\frac{1 + 3}{4 - 2}$ .

**Solution:** The fraction bar is a grouping symbol because it groups the terms on top and bottom of the bar. In this example, the fraction bar means the same as if you wrote the problem like the following:  $(1 + 3) \div (4 - 2)$ . You must simplify what's in parentheses first before you divide.

$$\frac{1 + 3}{4 - 2}$$

$$\frac{4}{2} = 2$$

### Radical Symbol

Any expression under a radical symbol is considered grouped, so the radical symbol is also a grouping symbol. The expression under the radical must be simplified before the root is taken. The root must be taken before any other operations can be performed. **Note:** When you take the square root of a number, the resulting root can be positive or negative. For example, both  $3 \cdot 3$  and  $(-3) \cdot (-3)$  equal 9. Therefore, the square root of 9 can be  $+3$  or  $-3$ . In the examples below, you will see only the positive root. We'll look at the negative root in later sections.

**Example 4:** Simplify the expression  $2 + \sqrt{25 - 16}$ .

**Step 1:** The square root of "25 minus 16" is a radical number that must be simplified first. When you subtract these two terms, the result is 9.

$$2 + \sqrt{25 - 16}$$

**Step 2:** Take the square root of 9. You probably recognize that 9 can be factored into  $3 \cdot 3$ , so the square root of 9 is 3.

$$2 + \sqrt{9}$$

**Step 3:** Finally, do the addition which gives a final answer of 5.

$$2 + 3 = 5$$



**Section 2.3, continued**  
**Order of Operations**  
**with Grouping Symbols**

**Substitution**

Now put all these skills together by substituting values into different types of expressions.

**Example 5:** What is the value of  $\frac{a^2 + 3}{2a}$  for  $a = -3$ ?

- |   |                            |
|---|----------------------------|
| <b>Step 1:</b> To simplify this expression, you must first substitute $-3$ for each $a$ in the expression. The fraction bar in this example is a grouping symbol, so you must simplify the top and bottom parts of the fraction separately before you divide. | $\frac{(-3)^2 + 3}{2(-3)}$ |
| <b>Step 2:</b> In the top part of the fraction, the exponent must be simplified first. In the bottom part of the fraction, the only operation is multiplication.  | $\frac{9 + 3}{-6}$         |
| <b>Step 3:</b> After the exponent is simplified, then you can add.  | $\frac{12}{-6}$            |
| <b>Step 4:</b> Once the top and the bottom of the fraction are completely simplified, you can divide.   | $-2$                       |

**Example 6:** What is the value of the expression below when  $a = 1$ ,  $b = -5$ , and  $c = 4$ ?

$$-b - \sqrt{b^2 - 4ac}$$

- |   |                                   |
|---|-----------------------------------|
| <b>Step 1:</b> Substitute in the values for $a$ , $b$ , and $c$ .   | $-(-5) - \sqrt{(-5)^2 - 4(1)(4)}$ |
| <b>Step 2:</b> When you substitute, there will be several sets of grouping symbols. You follow the left hand rule and simplify the first set of grouping symbols, the parentheses, first. The opposite of $-5$ is $5$ . | $5 - \sqrt{(-5)^2 - 4(1)(4)}$     |
| <b>Step 3:</b> The radical symbol is also a grouping symbol, so you must simplify everything under the radical next. Start with the exponent. $(-5)^2$ is $25$ .  | $5 - \sqrt{25 - 4(1)(4)}$         |
| <b>Step 4:</b> Next, you multiply under the radical.  | $5 - \sqrt{25 - 16}$              |
| <b>Step 5:</b> You can then do the subtraction under the radical.   | $5 - \sqrt{9}$                    |
| <b>Step 6:</b> Then, take the square root to finish simplifying the radical grouping symbol.  | $5 - 3$                           |
| <b>Step 7:</b> Finally, you subtract.   | $2$                               |

**Section 2.3, continued**  
**Order of Operations**  
**with Grouping Symbols**

**Practice 1**

Simplify the following expressions using the correct order of operations. Show your work, and write your final answer in the blank.

_____ 1. $12 \times (3 - 4) - 2^3$	_____ 2. $10 - (7 - 9)^2 \times 2$	_____ 3. $(3 - 1)^2 + 7$
_____ 4. $ 5 + 2  \times  3 - 6 $	_____ 5. $\frac{1 - 7}{3 + 3} \times -2$	_____ 6. $\frac{-2 \times \sqrt{6 + 3}}{3 \times 2}$

**Practice 2**

Simplify the following expressions for the values of  $x = 0$ ,  $y = -2$ , and  $z = 5$ . Show your work, and write your final answer in the blank.

_____ 1. $(x + 3) \cdot 2 + z$	_____ 2. $\frac{y^2 + 4}{2} + 3$	_____ 3. $3 \times (z + 2y)^2$
_____ 4. $(z + 2)^2$	_____ 5. $y^2 - \sqrt{-9y - 2}$	_____ 6. $ 3y - z + 2  - 4$
_____ 7. $y + z - \sqrt{z - 1}$	_____ 8. $z^2 + 2z - 5$	_____ 9. $\frac{z \cdot (y + 3)}{z + 2 \cdot y}$

# Properties of Real Numbers

## Section 2 Review

Answer each question below. Darken the circle that represents the correct answer.

1. Which of the following is equivalent to the equation  $2(x - 3) = 5(x + 1)$ ?

- A  $2x - 3 = 5x + 1$
- B  $2x - 6 = 5x$
- C  $2x - 6 = 5x + 5$
- D  $2x - 3 = 5x$

(A) (B) (C) (D)

4. Which of the following is equivalent to the expression below when  $a = -2$ ?

$$a^2 + 5a - 7$$

- A -19
- B -13
- C -1
- D 7

(A) (B) (C) (D)

2. If  $a = 0$ , which of the following expressions has a value of 0?

- A  $a \cdot (1 - 3 + 2)$
- B  $(a + 2) \cdot (4 - 3)$
- C  $a + (2 \cdot 3 + 4)$
- D  $(a - 4) \cdot 3 + 2$

(A) (B) (C) (D)

5. Which of the following is equivalent to

$$-b - \sqrt{b^2 - 4ac}$$

when  $a = 4$ ,  $b = -5$ , and  $c = 1$ ?

- A -4
- B 2
- C 3
- D 4

(A) (B) (C) (D)

3. Which of the following is equivalent to the expression below when  $x = 5$ ?

$$x^2 - 3x + 2$$

- A -3
- B 8
- C 12
- D 42

(A) (B) (C) (D)

6. What is the value of  $\frac{x+4}{x^2-1}$  for  $x = -1$ ?

- A  $-\frac{3}{2}$
- B  $\frac{3}{2}$
- C 0
- D undefined

(A) (B) (C) (D)

Section 2 Review, continued

7. Which of the following is equivalent to  $4.5(a + 3)$ ?

- A  $3a + 4.5$
- B  $4.5a + 4.5(3)$
- C  $4.5 + a + 3$
- D  $4.5a + 3$

(A) (B) (C) (D)

10. Which of the following is equivalent to the expression below when  $x = 3$ ?

$$-x^2 + \sqrt{4x - 8}$$

- A 13
- B 11
- C -7
- D -4

(A) (B) (C) (D)

8. Which of the following is equivalent to  $3.2(1.1 - x)$ ?

- A  $3.2(1.1) - x$
- B  $3.2 + 1.1 - x$
- C  $3.2(1.1) - 3.2x$
- D  $3.2 + 1.1 - 3.2x$

(A) (B) (C) (D)

11. Which of the following is equivalent to the expression below when  $x = -2$ ?

$$|3x + 4| - x$$

- A 0
- B 4
- C 8
- D 12

(A) (B) (C) (D)

9. Which of the following is equivalent to the equation below?

$$3(x + 4) = -(2 - x)$$

- A  $3x + 12 = -2 + x$
- B  $3x + 4 = -2x + x$
- C  $3x + 12 = -2 - x$
- D  $3x + 4 = 2 - x$

(A) (B) (C) (D)

12. If  $x$  is a negative integer, and  $y$  is equal to  $x$ , which of the following expressions will have a value of zero?

- A  $y - x$
- B  $x + y$
- C  $xy$
- D  $\frac{x}{y}$

(A) (B) (C) (D)