

When we encounter limits that involve radical expressions such as $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$, it is possible to determine the limit by rationalizing the numerator and denominator.

Recall from previous courses, that we rationalize an expression by multiplying by a conjugate. In this particular example, we would multiply $\frac{\sqrt{x}-1}{x-1}$ by $\frac{\sqrt{x}+1}{\sqrt{x}+1}$ and get the expression $\frac{x-1}{(x-1)(\sqrt{x}+1)}$.

This multiplication should provide terms that will cancel, thus allowing us to calculate the limit with direct substitution.

$$\text{Hence the } \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}.$$

Try to calculate the following:

$$\text{Example 1: } \lim_{x \rightarrow 7} \frac{\sqrt{x}-7}{x-7}$$

$$\text{Example 2: } \lim_{x \rightarrow 100} \frac{5x-500}{\sqrt{x}-10}$$

1. $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

2. $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$

3. $\lim_{x \rightarrow 6} \frac{\sqrt{x}-36}{x-6}$

4. $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{4x-16}$

5. $\lim_{x \rightarrow 16} \frac{3x-48}{\sqrt{x}-4}$

6. $\lim_{x \rightarrow 9} \frac{3-\sqrt{x}}{9-x}$

7. $\lim_{x \rightarrow 25} \frac{4x-100}{\sqrt{x}-5}$

8. $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$

9. $\lim_{x \rightarrow 5} \frac{\sqrt{x}-\sqrt{5}}{x-5}$

10. $\lim_{x \rightarrow 7} \frac{\sqrt{x}-\sqrt{7}}{x-7}$