

**NEW MILFORD PUBLIC SCHOOL**  
New Milford, Connecticut



**Intermediate Algebra II**

May 2012

*Approved by the Board of Education  
June 12, 2012*

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## **New Milford's Mission Statement**

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.

## **Intermediate Algebra II**

After a review of Algebra I, students will study selected topics from Algebra II including factoring polynomials, quadratic equations, rational expressions and equations, complex fractions, graphs of quadratic and absolute value functions, and appropriate word problems. Calculators and/or computers will be used. A scientific calculator is required of all students in this course.

**Pacing Guide**  
(based on a block schedule)

<b>Unit #</b>	<b>Title</b>	<b>Days</b>	<b>Pages</b>
1	Fundamental Concepts of Algebra	16	7 – 11
2	Functions	16	12 – 17
3	Systems of Linear Equations and Inequalities	10	18 – 23
4	Exponents and Polynomial Expressions	8	24 – 29
5	Factoring and Quadratic Equations	13	30 – 35
6	Rational Expressions	12	36 – 40
	Final Course Assessments		41
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Common Core State Standards for Mathematics  
*Mathematics Standards for High School*

**Key for the Standards**

**Number and Quantity**

N-RN	The Real Number System
N-Q	Quantities
N-CN	The Complex Number System
N-VM	Vector and Matrix Quantities

**Algebra**

A-SSE	Seeing Structure in Expressions
A-APR	Arithmetic with Polynomials and Rational Expressions
A-CED	Creating Equations
A-REI	Reasoning with Equations and Inequalities

**Functions**

F-IF	Interpreting Functions
F-BF	Building Functions
F-LE	Linear, Quadratic, and Exponential Models
F-TF	Trigonometric Functions

**Geometry**

G-CO	Congruence
G-SRT	Similarity, Right Triangles, and Trigonometry
G-C	Circles
G-GPE	Expressing Geometric Properties with Equations
G-GMD	Geometric Measurement and Dimension
G-MG	Modeling with Geometry

**Statistics and Probability**

S-ID	Interpreting Categorical and Quantitative Data
S-IC	Making Inferences and Justifying Conclusions
S-CP	Conditional Probability and the Rules of Probability
S-MD	Using Probability to Make Decisions

\*\* Portions of standards that appear in italics will not be addressed in the Intermediate Algebra 2 course. Students will master these topics during the Academic Algebra 2 course, which follows Intermediate Algebra 2.

## New Milford Public Schools

Committee Member: Colleen Peterson & Linda Scoralick Unit 1: Fundamental Concepts of Algebra	Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days : 16
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"> <li>• CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</li> <li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li> <li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li> <li>• CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></li> <li>• CC.9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</li> <li>• CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</li> <li>• CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</li> </ul>	
<b>Enduring Understandings</b> Generalizations of desired understanding via essential questions (Students will understand that ...)	<b>Essential Questions</b> Inquiry used to explore generalizations
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Algebra provides a way to use numbers, symbolic notation, and arithmetic operations to model, transform, simplify, and solve problems efficiently.</li> <li>• Information may be represented by physical models, diagrams, data tables, graphs, and symbolic expressions. Algebra facilitates correlation among the different representations, which may give different insights into the solution of a problem.</li> </ul>	<ul style="list-style-type: none"> <li>• How can algebra be used to model real world situations and scenarios?</li> <li>• How are algebraic operations and notation used to simplify algebraic and numeric expressions?</li> <li>• How are algebraic operations and notation used to solve equations and inequalities?</li> <li>• How do single variable equations and inequalities apply to real world situations?</li> <li>• How do literal equations apply to real world situations?</li> </ul>

<ul style="list-style-type: none"> <li>• Values, expressions, and polynomials can be simplified using specific processes.</li> <li>• Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> <li>• Literal equations can represent many real world situations.</li> <li>• Absolute value represents a positive distance from zero on a number line and may result in two solutions.</li> <li>• Teacher reviews graphing on the rectangular coordinate system and labeling points.</li> </ul>	
<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>• How to identify and solve single variable equations and inequalities, as well as literal and absolute value equations</li> <li>• How to translate algebraic expressions into words and written sentences into algebraic sentences</li> <li>• Key terms: literal equation, absolute value, expression, equation, inequality, evaluate, like terms</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>• Identify and know how to simplify algebraic expressions</li> <li>• Apply the rules of order of operations to evaluate numeric expressions</li> <li>• Identify and know how to solve single variable, literal, and absolute value equations</li> <li>• Identify and know how to solve single variable inequalities</li> <li>• Solve application problems using the methods listed above</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>• Cooperation</li> <li>• Honesty</li> <li>• Integrity</li> <li>• Perseverance</li> <li>• Respect</li> <li>• Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>• Show graphic representation of data.</li> <li>• Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>• Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	



## Develop Teaching and Learning Plan

### Suggested Teaching Strategies:

- Teacher checks for prior knowledge using common formative assessment (pre-test).
- Teacher checks for prerequisite knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.
- Teacher encourages higher order thinking skills through the use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a current topic and prior/prerequisite knowledge and/or a real world situation.
- Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.
- Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging, and which skills and concepts they feel need more practice in order to be mastered.
- Teacher models proper techniques and a variety of techniques for solving problems.
- Teacher models for students how to show complete work, formalize answers, check solutions (by hand and using technology).
- Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.

### Suggested Learning Activities:

- Students will use the graphing calculator to evaluate an algebraic expression.
- Students will check answers using a variety of methods, including using the graphing calculator to check solutions to equations.
- Students will review the concept of absolute value as it relates to the number line. Distance interpretation is as an option to solving absolute value equations.
- Students will practice/apply skills and concepts for solving equations in a variety of groupings including whole class, individual, and small groups. Students will have the opportunity to practice skills and concepts in each grouping situation during class period.
- Students will verbally explain processes used to simplify expressions and solve equations.
- Students will use white boards to practice evaluating expressions, simplifying expressions, and solving equations.

<ul style="list-style-type: none"> <li>Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of concepts.</li> </ul>	
<b>Assessments</b>	
<b>Performance Task</b> Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	<b>Other Evidence</b> Application that is functional in a classroom context to evaluate student achievement of desired results
<p><b>Goal:</b> To explore uses of equations in the real-world</p> <p><b>Role:</b> Politician</p> <p><b>Audience:</b> Law enforcement agents</p> <p><b>Situation:</b> The state government has just decided to change the way fines for speeding will be calculated. Students will be given the new fee schedule and asked to use the information to prepare a document that can be used to explain the new procedure for determining fines and highlight key features.</p> <p><b>Product:</b> One sheet document to be used by police officers/judges</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<ul style="list-style-type: none"> <li>Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>Quizzes</li> <li>Test (may include approximately 10 multiple-choice and 15-25 short answer questions)</li> </ul>

## Suggested Resources

- Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class.
  - Operations with positive and negative numbers
  - Evaluating expressions and order of operations
  - Simplifying expressions and the distributive property
  - The Coordinate Plane – plotting points, naming points, giving location of points, etc
  - Solving equations (including equations with fractions)
  - Solving proportions
  - Solving word problems: integer/age, mixture, coin
  - Solving literal equations
  - Solving absolute value equations
  - Solving inequalities
- [www.algebrafunsheets.com](http://www.algebrafunsheets.com) – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.
- [www.kutasoftware.com](http://www.kutasoftware.com) – Can be used to generate additional practice depending on needs of class or individual students.
- Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. *Algebra 2*. Upper Saddle River, NJ: Prentice Hall, 2009. Print.
- Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. *Algebra 1*. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print.

## New Milford Public Schools

Committee Members: Colleen Peterson & Linda Scoralick Unit 2: Functions	Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days: 16
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"><li>• CC.9-12.N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</li><li>• CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.</li><li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li><li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li><li>• CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</li><li>• CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</li><li>• CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</li><li>• CC.9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</li><li>• CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</li><li>• CC.9-12.F.IF.7a Graph linear and quadratic functions <i>and show intercepts, maxima, and minima</i>.</li></ul>	

<p style="text-align: center;"><b>Enduring Understandings</b></p> <p style="text-align: center;">Generalizations of desired understanding via essential questions (Students will understand that ...)</p>	<p style="text-align: center;"><b>Essential Questions</b></p> <p style="text-align: center;">Inquiry used to explore generalizations</p>
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Information may be represented by physical models, diagrams, data tables, graphs, and symbolic expressions. Algebra facilitates correlation among the different representations, which may give different insights into the solution of a problem.</li> <li>• Algebra provides ways to describe and classify relationships and functions and use the classifications to derive models that have practical real-world applications.</li> <li>• Algebra provides a way to explore and understand the effects of parameter changes on any function and its various representations.</li> <li>• Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> <li>• Linear, quadratic, and absolute value functions have different shapes when graphed.</li> <li>• Intercepts have special meanings in graphs.</li> <li>• Linear equations can be represented in many different forms.</li> <li>• The slope of a line represents the rate of change for a specific relation.</li> <li>• Functions are special relations and have a domain and a range.</li> </ul>	<ul style="list-style-type: none"> <li>• What are the relationships between equations of functions and their graphs?</li> <li>• What types of real world problems can be modeled with a linear equation?</li> <li>• Why is slope useful?</li> <li>• Why do different functions have different graphs and behaviors?</li> </ul>

<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>• The formula for slope</li> <li>• How to write equations of lines given a variety of information</li> <li>• How to graph lines given various forms of equations</li> <li>• What makes a relation a function</li> <li>• What the graphs of linear, quadratic, and absolute value functions look like</li> <li>• Composition of functions</li> <li>• Key terms: intercept, slope, line, slope-intercept form, point-slope form, standard form, horizontal, vertical, function, domain, range, relation, function notation, vertical line test, parabola, vertex, axis of symmetry, x- and y-intercepts</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>• Graph various equations and functions using different methods, including making a t-table</li> <li>• Find the slope of a line given two points, an equation, or a graph</li> <li>• Graph a line written in slope-intercept form, point-slope form, and standard form</li> <li>• Graph horizontal and vertical lines</li> <li>• Graph linear inequalities</li> <li>• Write an equation of a line in slope-intercept form and point slope form</li> <li>• Write equations for horizontal and vertical lines</li> <li>• Identify whether a graph or equation is a function and find the domain and range of a function</li> <li>• Perform operations involving the composition of functions</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>• Cooperation</li> <li>• Honesty</li> <li>• Integrity</li> <li>• Perseverance</li> <li>• Respect</li> <li>• Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>• Show graphic representation of data.</li> <li>• Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>• Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	
<b>Develop Teaching and Learning Plan</b>	
<p>Suggested Teaching Strategies:</p> <ul style="list-style-type: none"> <li>• Teacher checks for prior knowledge using common formative assessment (pre-test).</li> <li>• Teacher checks for prerequisite</li> </ul>	<p>Suggested Learning Activities:</p> <ul style="list-style-type: none"> <li>• Students will use flash cards to practice identifying slope, y-intercept, and points from equations of lines written in point-slope and slope-</li> </ul>

<p>knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.</p> <ul style="list-style-type: none"> <li>• Teacher encourages higher order thinking skills through use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a current topic and prior/prerequisite knowledge and/or a real world situation.</li> <li>• Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.</li> <li>• Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging and which skills and concepts they feel need more practice in order to be mastered.</li> <li>• Teacher models proper techniques and a variety of techniques for solving problems.</li> <li>• Teacher models for students how to show complete work, formalize answers, and check solutions (by hand and by using technology).</li> <li>• Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.</li> <li>• Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of</li> </ul>	<p>intercept form.</p> <ul style="list-style-type: none"> <li>• Students will use white boards as a tool to help identify slope and y-intercept for equations written in standard form.</li> <li>• Students will use flash cards in pairs to practice identifying horizontal and vertical lines.</li> <li>• Students will match an equation of a line to the appropriate graph.</li> <li>• Students will brainstorm, in pairs, where they might see things with positive, negative, zero, and undefined slope in the real world.</li> <li>• Students will use white boards to practice graphing lines given slope-intercept, point-slope, and standards form.</li> <li>• Students will practice graphing equations of horizontal and vertical lines independently.</li> <li>• Students will work independently graphing functions using a t-table.</li> <li>• Students will work in small groups to practice reading and interpreting real world graphs.</li> <li>• Students will write equations of lines presented with a variety of information. Information may include, slope, point on the line, y-intercept, or two points.</li> <li>• Students will participate in whole class and small group discussions and activities to use data from real world situations to create/interpret graphs, create linear models, and make predictions.</li> <li>• Students will work in pairs using flash cards to identify domain and range from discrete relations. Students will also determine if these discrete relations are functions.</li> <li>• Students will practice expressing relations in a variety of formats including: set of ordered pairs, table, discrete graph, and mapping diagrams.</li> </ul>
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<p>concepts.</p> <ul style="list-style-type: none"> <li>• Teacher models graphing function using a t-table (graphic organizer) and allows students class time to practice graphing various functions.</li> <li>• Teacher models graphing linear equations from slope-intercept and point-slope form, as well as horizontal and vertical lines, using methods and processes specific to these forms of equations.</li> <li>• Teacher introduces the concept of slope and discusses different types of slope (positive, negative, zero, and undefined).</li> <li>• Teacher models how to use the slope formula.</li> <li>• Teacher models how to find the slope of a line from a graph, from an equation, and given two points.</li> </ul>	
<b>Assessments</b>	
<p style="text-align: center;"><b>Performance Task</b></p> <p>Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)</p>	<p style="text-align: center;"><b>Other Evidence</b></p> <p>Application that is functional in a classroom context to evaluate student achievement of desired results</p>
<p><b>Goal:</b> To interpret real-world graphs, write equations from graphs, and make predictions using that equation</p> <p><b>Role:</b> Statistician</p> <p><b>Audience:</b> Managers of two companies</p> <p><b>Situation:</b> Students are given two graphs. One detailing the average cost of a retail prescription and the other detailing the number of workers per Social Security beneficiary. Students are to identify and interpret the y-intercept, slope, linear equation, and explain the meaning of these values.</p> <p><b>Product:</b> Student will write future predictions based on their findings.</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<ul style="list-style-type: none"> <li>• Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>• Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>• Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>• Quizzes</li> <li>• Tests(may include approximately 10 multiple-choice and 15-25 short answer questions) <ul style="list-style-type: none"> <li>○ It is recommended that graphing lines and writing equations of lines be tested separately.</li> </ul> </li> <li>• Review quiz on material mastered in Unit 1</li> </ul>



## Suggested Resources

- Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class.
  - Slope of a line and the slope formula
  - Slopes of parallel and perpendicular lines
  - Graphing lines using t-charts, slope-intercept form, and point-slope form
  - Graphing horizontal and vertical lines
  - Graphing inequalities
  - Writing equations of lines in slope-intercept form and point-slope form
  - Relations and functions/function notation
  - Evaluating functions
  - Graphing functions using t-charts
- [www.algebrafunsheets.com](http://www.algebrafunsheets.com) – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.
- [www.kutasoftware.com](http://www.kutasoftware.com) – Can be used to generate additional practice depending on needs of class or individual students.
- Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. *Algebra 2*. Upper Saddle River, NJ: Prentice Hall, 2009. Print.
- Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. *Algebra 1*. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print. Green Globbs – computer program

## New Milford Public Schools

<p>Committee Members: Colleen Peterson &amp; Linda Scoralick Unit 3: Systems of Linear Equations and Inequalities</p>	<p>Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days: 10</p>
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"> <li>• CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</li> <li>• CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li> <li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li> <li>• CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</li> <li>• CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</li> </ul>	
<p><b>Enduring Understandings</b> Generalizations of desired understanding via essential questions (Students will understand that ...)</p>	<p><b>Essential Questions</b> Inquiry used to explore generalizations</p>
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Algebra provides a way to use numbers, symbolic notation, and arithmetic operations to model, transform, simplify, and solve problems efficiently.</li> <li>• Information may be represented by physical models, diagrams, data tables, graphs, and symbolic expressions. Algebra facilitates correlation among the different representations, which may give different insights into the solution of a problem.</li> <li>• Values, expressions, and polynomials can be simplified using specific processes.</li> </ul>	<ul style="list-style-type: none"> <li>• What is the significance of a solution to a system of equations or inequalities?</li> <li>• How are the algebraic methods (substitution, elimination) related to solving systems of equations by graphing?</li> <li>• How can systems of equations be used to help solve problems in the real world?</li> </ul>

<ul style="list-style-type: none"> <li>Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> </ul>	
<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>The solution to a system of linear equations represents the intersection of two lines in the coordinate plane</li> <li>The solution to a system of linear inequalities is a region in the coordinate plane</li> <li>How to identify and solve systems of linear equations using three methods: graphing, substitution, and elimination</li> <li>How to identify and solve a system of linear inequalities by graphing</li> <li>How to write systems of equations that can be used to solve application problems</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>Solve systems of linear equations using three different methods: graphing, substitution, and elimination</li> <li>Examine the characteristics of a system of linear equations and determine the most efficient method of solving</li> <li>Solve systems of linear inequalities using the graphing methods</li> <li>Solve real-world and verbal problems using the methods listed above</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>Cooperation</li> <li>Honesty</li> <li>Integrity</li> <li>Perseverance</li> <li>Respect</li> <li>Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>Show graphic representation of data.</li> <li>Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	

## Develop Teaching and Learning Plan

### Suggested Teaching Strategies:

- Teacher checks for prior knowledge using common formative assessment (pre-test).
- Teacher checks for prerequisite knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.
- Teacher encourages higher order thinking skills through the use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a current topic and prior/prerequisite knowledge and/or a real world situation.
- Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.
- Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging, and which skills and concepts they feel need more practice in order to be mastered.
- Teacher models proper techniques and a variety of techniques for solving problems.
- Teacher models for students how to show complete work, formalize answers, and check solutions (by hand and by using technology).
- Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.

### Suggested Learning Activities:

- Students will work independently and as a class solving systems of linear equations by graphing, by substitution, and by elimination.
- Students will verbally describe the steps that should be taken to solve a system by graphing, by substitution, and by elimination.
- Students will use flash cards to practice identifying a solution to a system that has been solved graphically.
- Students will explain how to use a graph to solve a system of equations.
- Students will use white boards to practice solving systems of equations and inequalities by graphing.
- Students will practice, in pairs, using systems of equations to model real-world situations.
- Students will independently practice analyzing the characteristics of a system of equations and determine the most efficient way to solve. They will be able to justify their decision in writing.
- Students will participate in a stations activity to review solving systems of equations and using systems to solve real world problems. Each station will help students review a different skill/concept from the unit.
- Students will compare and contrast the three methods of solving systems of linear equations, noting the advantages and disadvantages of each method.

- Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of concepts.
- Teacher describes why solving systems of equations by graphing works. Teacher explains that each point on a line represents an x- and y-value that makes the equation of the line true and that the point of intersection for the two lines represents an x-value and a y-value that work in both equations.
- Teacher asks students to explain the concept described above in their own words both in writing and verbally.
- Teacher explains the connection between the algebraic methods of solving and the point of intersection of two lines when solving graphically.
- Teacher models for students how to solve using the substitution and elimination methods, including how to show work, formalize answers (as ordered pairs), and check their solutions.
- Teacher allows students time in class, in a variety of different grouping structures, to practice solving using all three methods.
- Teacher models for students how to write and to use a system of equations in order to solve a real-world problem.
- Teacher models for students how to choose the most efficient method for solving a system of equations.
- Teacher draws connections between the process of graphing the boundary line for inequalities and the process of graphing lines.

<ul style="list-style-type: none"> <li>• Teacher models for students how to determine whether to use a solid or a dotted boundary line when graphing a linear inequality.</li> <li>• Teacher models for students how to determine which side of a line to shade for a linear inequality. The use of two to differentiate the solution areas for each inequality and more clearly see the solution area is recommended.</li> <li>• Teacher models for students how to mark the solution area for a system of linear inequalities.</li> </ul>	
<b>Assessments</b>	
<p style="text-align: center;"><b>Performance Task</b></p> <p style="text-align: center;">Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)</p>	<p style="text-align: center;"><b>Other Evidence</b></p> <p style="text-align: center;">Application that is functional in a classroom context to evaluate student achievement of desired results</p>
<p><b>Goal:</b> To use systems of linear equations to solve an application problem</p> <p><b>Role:</b> Researcher</p> <p><b>Audience:</b> Landline Phone Company</p> <p><b>Situation:</b> Students will be given data for the number of landline and cell phone customers for the years 2000 through 2006. They will be asked to use this data to make a graph, write an algebraic model for each type of phone line, find the year in which the usage was the same for both types of phone line, determine how many customers used each type of phone line in that year, and predict when the number of landline customers will be zero. Students will then be asked to write a short summary of their findings.</p> <p><b>Products:</b> Graph, models, and calculations to find year/number of customers when the two types of phone lines have equal usage, and a summary of results</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<ul style="list-style-type: none"> <li>• Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>• Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>• Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>• Quizzes</li> <li>• Test (may include approximately 10 multiple-choice and 15-25 short answer questions)</li> <li>• Review quiz on material mastered in Units 1 and 2</li> </ul>

## Suggested Resources

- Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class.
  - Systems by graphing, substitution, elimination
  - Systems of inequalities
  - Word Problems: integer/age and cost
- [www.algebrafunsheets.com](http://www.algebrafunsheets.com) – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.
- [www.kutasoftware.com](http://www.kutasoftware.com) – Can be used to generate additional practice depending on needs of class or individual students.
- Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. *Algebra 2*. Upper Saddle River, NJ: Prentice Hall, 2009. Print.
- Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. *Algebra 1*. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print.

## New Milford Public Schools

Committee Members: Colleen Peterson & Linda Scoralick Unit 4: Exponents and Polynomial Expressions	Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days: 8
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"> <li>• CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</li> <li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li> <li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li> <li>• CC.9-12.A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression <math>1.15^t</math> can be rewritten as <math>[1.15^{(1/12)}]^{(12t)} \approx 1.012^{(12t)}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</li> <li>• CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</li> </ul>	
<b>Enduring Understandings</b> Generalizations of desired understanding via essential questions (Students will understand that ...)	<b>Essential Questions</b> Inquiry used to explore generalizations
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Values, expressions, and polynomials can be simplified using specific processes.</li> <li>• Polynomials can be added, subtracted, multiplied, and divided in order to write the expression in a more simplified form.</li> <li>• Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> </ul>	<ul style="list-style-type: none"> <li>• How are the properties of exponents related to the basic arithmetic operations?</li> <li>• How are polynomial expressions combined using operations of addition, subtraction, multiplication, and division?</li> </ul>



<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>• Properties of exponents</li> <li>• Degree of a polynomial</li> <li>• Processes for addition, subtraction, multiplication, and division of polynomials</li> <li>• Key terms: exponent, polynomial, degree, term, power, monomial</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>• Simplify expressions using the properties of exponents</li> <li>• Classify polynomials based on degree and number of terms</li> <li>• Simplify, add, subtract, multiply, and divide monomials and polynomials</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>• Cooperation</li> <li>• Honesty</li> <li>• Integrity</li> <li>• Perseverance</li> <li>• Respect</li> <li>• Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>• Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>• Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	
<b>Develop Teaching and Learning Plan</b>	
<p>Suggested Teaching Strategies:</p> <ul style="list-style-type: none"> <li>• Teacher checks for prior knowledge using common formative assessment (pre-test).</li> <li>• Teacher checks for prerequisite knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.</li> <li>• Teacher encourages higher order thinking skills through the use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a</li> </ul>	<p>Suggested Learning Activities:</p> <ul style="list-style-type: none"> <li>• Students will work independently and as a class simplifying expressions with exponents.</li> <li>• Students will practice simplifying basic expressions with exponents, in pairs, using flash cards.</li> <li>• Students will participate in an activity to match multiplication problems with their simplified product.</li> <li>• Students will work in pairs to review sample solutions for errors. They will explain the errors and how to correct them.</li> <li>• Students will verbally explain the process for multiplying two binomials.</li> <li>• Students will explain, in writing, why it is not possible to square a sum or difference by simply squaring each</li> </ul>

current topic and prior/prerequisite knowledge and/or a real world situation.

- Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.
- Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging, and which skills and concepts they feel need more practice in order to be mastered.
- Teacher models proper techniques and a variety of techniques for solving problems.
- Teacher models for students how to show complete work, formalize answers, check solutions (by hand and by using technology).
- Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.
- Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of concepts.
- Teacher introduces students to the process used to classify polynomials based on degree and number of terms. Stress that this information will be required in order to understand what is said/written throughout the rest of the course and in future courses
- Teacher introduces the properties of exponents as a set of short cuts to what exponents mean. As more properties are introduced, more complex problems should be given to simplify as a class.

term of the sum or difference. They will also be able to do the same for sum/difference to other powers.

- Students will use white boards to practice dividing polynomials.
- Students will write quiz questions and trade with a partner. They will then solve the problems written by their partner. This activity can be applied to all concepts from this unit.

- Teacher models for students the questions they should ask themselves to help determine which property to be used.
- Teacher breaks down the rules to show where they come from and how the more complicated rules are extensions of the basic properties.
- Teacher illustrates why the zero power always results in “one” by showing repeated division from a fourth power, to a third power, to a second power, etc.
- Teacher extends the concept described above to illustrate how to simplify negative exponents.
- Teacher reinforces the concepts of the zero power and negative exponents using the property for dividing powers.
- Before introducing more complicated exponential expressions, teacher reviews the order of operations, highlighting the fact that exponents must be done before multiplication.
- Teacher allows ample time for practice both in and out of class and over the course of several class days. Material should be reviewed through warm-ups and on homework assignments.
- Teacher helps students recall the rules for combining like terms and summarizes students’ thoughts. Teacher stresses that when combining like terms only the coefficient changes, while the base and power remain the same.
- Teacher models addition and subtraction of polynomials, reminding students that subtraction is like distributing a negative one and then adding.
- Teacher encourages students to circle the sign between the sets of parentheses.
- Teacher models for students ways of visually differentiating like terms

within a string a terms. This may include use of different colors or shapes to mark like terms.

- Teacher models for students alternate ways of organizing polynomials and showing work (i.e., vertical structure for addition).
- Teacher reviews the distributive property.
- Teacher connects the distributive property with the concept of multiplying polynomials by explaining how the distributive property is used to multiply a monomial by a polynomial of two or more terms.
- Teacher utilizes the acronym F.O.I.L when teaching students to multiply two binomials and models for students ways of visually highlighting which terms are to be multiplied next.
- Teacher explains the connection between F.O.I.L. and the distributive property.
- Teacher models for students how multiplying polynomials with more than two terms each utilizes the distributive property.
- Teacher differentiates instruction by modeling for students different ways of showing work when multiplying polynomials with varying numbers of terms.
- Teacher provides ample time, in class, for students to practice, both individually and in small groups, each type of multiplication before introducing more complicated problems.
- Teacher helps students recall what it means to square a quantity or raise a quantity to the third power. Teacher highlights the difference between taking a sum/difference to a power and the product to a power property.
- Teacher draws connections between rules for dividing powers

<p>and dividing monomials.</p> <ul style="list-style-type: none"> <li>• Teacher models for students how to break up division problems into a series of fractions and to simplify each fraction separately.</li> </ul>	
<b>Assessments</b>	
<b>Performance Task</b>	<b>Other Evidence</b>
<p>Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)</p>	<p>Application that is functional in a classroom context to evaluate student achievement of desired results</p>
<p><b>Goal:</b> To identify correct and incorrect steps for simplifying polynomial expressions</p> <p><b>Role:</b> Teacher</p> <p><b>Audience:</b> Student who solved the problem</p> <p><b>Situation:</b> Students are given a problem set with specific steps shown as a solution. They then identify if each step is correct or incorrect, for each problem, and explain why.</p> <p><b>Product:</b> Corrected problem to include feedback and explanation</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<ul style="list-style-type: none"> <li>• Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>• Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>• Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>• Quizzes</li> <li>• Test (may include approximately 10 multiple-choice and 15-25 short answer questions)</li> <li>• Review quiz on material mastered in Units 1 through 3</li> </ul>
<b>Suggested Resources</b>	
<ul style="list-style-type: none"> <li>• Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class. <ul style="list-style-type: none"> <li>○ Polynomial vocabulary</li> <li>○ Exponent rules</li> <li>○ Operations with polynomials: adding, subtracting, multiplying, dividing</li> </ul> </li> <li>• <a href="http://www.algebrafunsheets.com">www.algebrafunsheets.com</a> – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.</li> <li>• <a href="http://www.kutasoftware.com">www.kutasoftware.com</a> – Can be used to generate additional practice depending on needs of class or individual students.</li> <li>• Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. <i>Algebra 2</i>. Upper Saddle River, NJ: Prentice Hall, 2009. Print.</li> <li>• Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. <i>Algebra 1</i>. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print.</li> </ul>	

## New Milford Public Schools

Committee Members: Colleen Peterson & Linda Scoralick Unit 5: Factoring and Quadratic Equations	Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days: 13
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"> <li>• CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>• CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</li> <li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li> <li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li> <li>• CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>• CC.9-12.A.REI.4 Solve quadratic equations in one variable.</li> <li>• CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking <i>square roots</i>, <i>completing the square</i>, the quadratic formula and factoring, as appropriate to the initial form of the equation. <i>Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</i></li> </ul>	
<b>Enduring Understandings</b> Generalizations of desired understanding via essential questions (Students will understand that ...)	<b>Essential Questions</b> Inquiry used to explore generalizations
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Algebra provides a way to use numbers, symbolic notation, and arithmetic operations to model, transform, simplify, and solve problems efficiently.</li> <li>• Algebra provides ways to describe and classify relationships and functions and use the classifications to derive models that have practical real-world applications.</li> <li>• Polynomials can be broken up into products of more simplified terms</li> </ul>	<ul style="list-style-type: none"> <li>• How are polynomial expressions combined using operations of addition, subtraction, multiplication, and division?</li> <li>• How can quadratic equations be solved?</li> <li>• How do quadratic functions relate to real-world situations?</li> </ul>

<p>by factoring.</p> <ul style="list-style-type: none"> <li>• Quadratic equations can be solved using a variety of methods, including factoring and the quadratic formula.</li> <li>• Quadratic functions are shaped like parabolas and have special properties.</li> </ul>	
<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>• Processes and steps used to factor polynomials</li> <li>• The methods and processes that can be used to solve quadratic equations by factoring and the quadratic formula</li> <li>• Key terms: factor, quadratic formula, polynomial</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>• Examine the characteristics of a polynomial and factor it using the following methods: greatest common factor, grouping, master product, difference of squares, sum/difference of cubes</li> <li>• Solve quadratic equations by factoring or using the quadratic formula</li> <li>• Solve real-world and verbal problems using the methods listed above</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>• Cooperation</li> <li>• Honesty</li> <li>• Integrity</li> <li>• Perseverance</li> <li>• Respect</li> <li>• Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>• Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>• Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	
<b>Develop Teaching and Learning Plan</b>	
<p>Suggested Teaching Strategies:</p> <ul style="list-style-type: none"> <li>• Teacher checks for prior knowledge using common formative assessment (pre-test).</li> <li>• Teacher checks for prerequisite knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.</li> </ul>	<p>Suggested Learning Activities:</p> <ul style="list-style-type: none"> <li>• Students will use white boards to practice factoring polynomial expressions.</li> <li>• Students will practice factoring polynomial expressions individually and in small groups.</li> <li>• Students will examine a given solution for a factoring problem to</li> </ul>

<ul style="list-style-type: none"> <li>• Teacher encourages higher order thinking skills through use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a current topic and prior/prerequisite knowledge and/or a real world situation.</li> <li>• Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.</li> <li>• Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging, and which skills and concepts they feel need more practice in order to be mastered.</li> <li>• Teacher models proper techniques and a variety of techniques for solving problems.</li> <li>• Teacher models for students how to show complete work, formalize answers, check solutions (by hand and by using technology).</li> <li>• Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.</li> <li>• Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of concepts.</li> <li>• Teacher models how to factor polynomial expressions with GCF and grouping one day, then</li> </ul>	<p>find errors. They will then explain, in writing, what is wrong, why it is wrong, and how it should be corrected.</p> <ul style="list-style-type: none"> <li>• Students will solve quadratic equations using the methods of factoring and the quadratic formula by working in pairs.</li> <li>• Students will verbally state the quadratic formula from memory and may use a song or story as a way of stating it.</li> <li>• Students will practice factoring quadratic trinomials with a leading coefficient of one using the “backwards F.O.I.L” method utilizing flash cards.</li> <li>• Students will work in pairs to generate and factor polynomials of varying types.</li> <li>• Students will write and solve quadratic equations to model real-world area problems.</li> </ul>
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trinomials (using the Master Product method), then difference of squares and sum/difference of cubes.

Teacher must allow many opportunities for small group practice in class and independent practice, both in and out of class.

- Teacher differentiates instruction by modeling for students the “backwards F.O.I.L” method for quadratic trinomials with a leading coefficient of one. Refer to this method as often as possible to help students see the connection between Master Product and this method.
- Teacher uses a flow chart to help students determine which method of factoring should be used to factor a polynomial and highlights for students that no matter what a polynomial looks like, the process of factoring always starts with factoring out a GCF if possible.
- Teacher reinforces the use of the flow chart by modeling the questions students should be asking themselves when examining a polynomial and attempting to factor.
- Teacher incorporates spiral review into each lesson via warm-up and homework assignments that review previously learned methods of factoring.
- Teacher scaffolds learning so that factoring problems become progressively more challenging, requiring students to factor more than once to completely factor a single polynomial.
- Teacher models for students how to check work and encourages them to multiply to check all factored polynomials. Teacher stresses what form the answer to a factoring question should be in and models ways of effectively organizing work.

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| <ul style="list-style-type: none"><li>• Teacher models how to solve quadratic equations by factoring and highlights the importance of moving all terms to one side before attempting to factor.</li><li>• Teacher explains why solving by factoring works using the Zero Product Property.</li><li>• Teacher illustrates for students how a graph of a quadratic function can be used to find the zeroes of the function. Teacher draws connections between these values and the solutions that would be found algebraically.</li><li>• Teacher discusses the need for having the quadratic formula as an alternative method for solving quadratic equations. Ample time must be devoted to allowing students to practice each method.</li><li>• Teacher uses graphs to visually show students how a quadratic equation can have zero, one, or two real solutions. Teacher connects these graphs with the different methods of solving.</li><li>• Teacher uses a song or a story as a way to memorize the quadratic formula.</li></ul> |  |
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<b>Assessments</b>	
<b>Performance Task</b>	<b>Other Evidence</b>
<p>Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)</p> <p><b>Goal:</b> To apply polynomial equations to a problem relating to construction of a swimming pool</p> <p><b>Role:</b> Builder</p> <p><b>Audience:</b> Client</p> <p><b>Situation:</b> The builder (student) has been asked to submit a bid to build a deck around a swimming pool. The student has the dimensions of the swimming pool and the homeowner has told the builder how much total area they would like taken up by both the pool and the deck. The builder must now determine the dimensions of the deck and write a proposal for the homeowner that will include time and materials.</p> <p><b>Product:</b> Completed proposal</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<p>Application that is functional in a classroom context to evaluate student achievement of desired results</p> <ul style="list-style-type: none"> <li>• Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>• Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>• Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>• Quizzes</li> <li>• Test (may include approximately 10 multiple-choice and 15-25 short answer questions)</li> <li>• Review quiz on material mastered in Units 1 through 4</li> </ul>
<b>Suggested Resources</b>	
<ul style="list-style-type: none"> <li>• Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class. <ul style="list-style-type: none"> <li>○ Factoring by GCF and grouping, quadratic trinomials, difference of squares, sum/difference of cubes</li> <li>○ Factoring mixed practice</li> <li>○ Solving quadratic equations by factoring</li> <li>○ Quadratic formula</li> <li>○ Solving word problems: integer and area</li> </ul> </li> <li>• <a href="http://www.algebrafunsheets.com">www.algebrafunsheets.com</a> – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.</li> <li>• <a href="http://www.kutasoftware.com">www.kutasoftware.com</a> – Can be used to generate additional practice depending on needs of class or individual students.</li> <li>• Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. <i>Algebra 2</i>. Upper Saddle River, NJ: Prentice Hall, 2009. Print.</li> <li>• Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. <i>Algebra 1</i>. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print.</li> </ul>	

## New Milford Public Schools

Committee Members: Colleen Peterson & Linda Scoralick Unit 6: Rational Expressions	Course/Subject: Intermediate Algebra II Grade Levels: 10 – 12 # of Days: 12
<b>Identify Desired Results</b>	
<b>Common Core Standards</b>	
<ul style="list-style-type: none"> <li>• CC.9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>• CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</li> <li>• CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</li> <li>• CC.9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</li> <li>• CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>• CC.9-12.A.APR.6 Rewrite simple rational expressions in different forms; write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</li> <li>• CC.9-12.A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</li> </ul>	
<b>Enduring Understandings</b> Generalizations of desired understanding via essential questions (Students will understand that ...)	<b>Essential Questions</b> Inquiry used to explore generalizations
<ul style="list-style-type: none"> <li>• The fundamental structure of algebra provides a systematic method for identifying, describing, extending, analyzing, and generalizing patterns.</li> <li>• Rational expressions can be simplified, added, subtracted, multiplied, and divided using the same concepts and procedures as with strictly numeric fractions.</li> <li>• Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> </ul>	<ul style="list-style-type: none"> <li>• How are polynomial expressions combined using operations of addition, subtraction, multiplication, and division?</li> <li>• How can rational expressions be simplified?</li> </ul>

<b>Expected Performances</b> What students should know and be able to do	
<p>Students will know the following:</p> <ul style="list-style-type: none"> <li>Processes used to simplify and perform operations with rational expressions</li> <li>Key terms: least common denominator, complex fraction, polynomial</li> </ul> <p>Students will be able to do the following:</p> <ul style="list-style-type: none"> <li>Simplify, add, subtract, multiply, and divide rational expressions</li> <li>Simplify complex fractions</li> </ul>	
<b>Character Attributes</b>	
<ul style="list-style-type: none"> <li>Cooperation</li> <li>Honesty</li> <li>Integrity</li> <li>Perseverance</li> <li>Respect</li> <li>Responsibility</li> </ul>	
<b>Technology Competencies</b>	
<ul style="list-style-type: none"> <li>Independently use appropriate technology tools to define problems and to propose hypothesis.</li> <li>Use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.</li> </ul>	
<b>Develop Teaching and Learning Plan</b>	
<p>Suggested Teaching Strategies:</p> <ul style="list-style-type: none"> <li>Teacher checks for prior knowledge using common formative assessment (pre-test).</li> <li>Teacher checks for prerequisite knowledge throughout the unit using warm-up problems, questioning activities, and spiral review problems.</li> <li>Teacher encourages higher order thinking skills through the use of math journals (exit tickets) and discussion. Journal prompts might include asking students to explain their approach to a problem, explain an alternate approach to a problem, how to check that their work was accurate, how to explain a concept to a student that was absent, or to draw connections between a current topic and prior/prerequisite knowledge and/or a real world situation.</li> </ul>	<p>Suggested Learning Activities:</p> <ul style="list-style-type: none"> <li>Students will verbally explain the process used to simplify, multiply, divide, add, and subtract rational expressions.</li> <li>Students will work independently and in small groups to practice simplifying rational expressions.</li> <li>Students will work in small groups on board exercises involving operations with algebraic fractions.</li> <li>Students will be able to determine and explain whether or not an expression is completely simplified and why.</li> <li>Students will be able to explain writing whether or not terms can be cancelled and why.</li> <li>Students will practice finding an LCD using flash cards.</li> <li>Students will compare the number of steps used in each method of simplifying complex fractions and</li> </ul>

- Teacher uses exit tickets and common formative assessments at the end of each lesson to guide planning for future lessons.
- Teacher encourages students to reflect on their own learning after each lesson (reflective journal via exit ticket). Prompts may include asking students what they found easy in a lesson, which parts of a lesson were the most challenging, and which skills and concepts they feel need more practice in order to be mastered.
- Teacher models proper techniques and a variety of techniques for solving problems.
- Teacher models for students how to show complete work, formalize answers, check solutions (by hand and by using technology).
- Teacher models and cues expected behaviors for appropriate classroom behavior including participation and note taking skills.
- Teacher uses a variety of grouping strategies (including whole class, individual, and small groups) to allow students varied opportunities to build strong foundational skills and deeper understanding of concepts.
- Teacher reminds students that when we talk about rational expressions, we are talking about fractions.
- Teacher draws connections between the processes used to simplify numeric fractions and algebraic fractions.
- Teacher models all operations with algebraic fractions with both numeric and already factored polynomial expressions.
- Teacher lists concrete and sequential steps for each operation to be performed with algebraic fractions.

make a case for which method is most efficient and why.

- Students will explain, in writing, how to find the least common denominator for two polynomial denominators.
- Students will conduct error analysis on sample solutions provided by the teacher.

<ul style="list-style-type: none"> <li>• Teacher models for students ways of organizing work to separate factoring scratch work from work to simplify fractions.</li> <li>• Teacher uses the phrase “Keep. Change. Flip.” to help students remember how to divide fractions.</li> <li>• Teacher models for students how division in complex fractions can be rewritten using the division symbol.</li> <li>• Teacher models two approaches for simplifying complex fractions: combine and simplify; multiply all fractions by the LCD.</li> <li>• Teacher provides ample time to practice each operation with algebraic fractions before moving on to the next operation.</li> </ul>	
<b>Assessments</b>	
<b>Performance Task</b> Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	<b>Other Evidence</b> Application that is functional in a classroom context to evaluate student achievement of desired results
<p><b>Goal:</b> To identify correct and incorrect steps for simplifying rational expressions</p> <p><b>Role:</b> Teacher</p> <p><b>Audience:</b> Student who solved the problem</p> <p><b>Situation:</b> Students are given a problem set with specific steps shown as a solution. Students then identify if each step is correct or incorrect and explain why.</p> <p><b>Product:</b> Corrected problem to include feedback and explanation</p> <p><b>Standard for Success:</b> Mathematics department scoring rubric</p>	<ul style="list-style-type: none"> <li>• Monitoring class work through board work, group work, questioning, and walk-arounds</li> <li>• Check for understanding via going over homework, board and white board activities, math journals, and reflective journals</li> <li>• Common Formative Assessments: pre-test, lesson exit tickets, and post-test</li> <li>• Quizzes</li> <li>• Test (may include approximately 10 multiple-choice and 15-25 short answer questions)</li> <li>• Review quiz on material mastered in Units 1 through 5</li> </ul>

## Suggested Resources

- Lesson worksheets: Samples are available in the math department. Lesson worksheets on the following topics should be included and should address the topics from each particular lesson in order to reinforce concepts and to provide students with additional practice for in and out of class.
  - Simplifying rational expressions
  - Multiplying, dividing, adding, and subtracting rational expressions
  - Complex fractions
- [www.algebrafunsheets.com](http://www.algebrafunsheets.com) – Can be used as a source of additional practice. Many topics will have practice in the form of puzzles which may help with student engagement.
- [www.kutasoftware.com](http://www.kutasoftware.com) – Can be used to generate additional practice depending on needs of class or individual students.
- Textbook: Bellman, Bragg, Charles, Hall, Handlin, Kennedy. *Algebra 2*. Upper Saddle River, NJ: Prentice Hall, 2009. Print.
- Textbook: Burger, Chard, Hall, Kennedy, Leinwand, Renfro, Seymour, Waits. *Algebra 1*. Austin, Texas: Holt, Reinhart, and Winston, 2007. Print.



## **Final Course Assessments**

The midyear assessment consists of approximately thirty-seven multiple choice forty-seven short answer questions which require work to be shown, an explanation, a graph, or a written response.

The final course assessment consists of approximately thirty-two multiple choice and fifty short answer questions which require work to be shown, an explanation, a graph, or a written response.

Rubrics are used for judging the success of major projects, papers, and presentations.

## Third Generation CAPT Scoring Rubric

### Score 3

The student has demonstrated a **full and complete** understanding of all concepts and processes essential to this application. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student's competence in problem-solving and reasoning, computing and estimating, and communicating to the full extent that these processes apply to the specified task. The response may, however, contain minor arithmetic errors that do not detract from a demonstration of full understanding. Student work is shown or an explanation is included.

### Score 2

The student has demonstrated a **reasonable** understanding of the essential mathematical concepts and processes in this application. The student's response contains most of the attributes of an appropriate response including a mathematically sound approach and evidence of competence with applicable mathematical processes but contains flaws that do not diminish the evidence that the student comprehends the essential mathematical ideas addressed in the task. Such flaws include errors attributed to faulty reading, writing, or drawing skills; errors attributed to insufficient, non-mathematical knowledge, and errors attributed to careless execution of mathematical processes or algorithms.

### Score 1

The student has demonstrated a **partial** understanding of some of the concepts and processes in this application. The student's response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge, errors in fundamental mathematical procedures, and other omissions or irregularities that bring into question the extent of the student's ability to solve problems of this general type.

### Score 0

The student has demonstrated **merely an acquaintance** with the topic. The student's response is associated with the task in the item but contains few attributes of an appropriate response. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical ideas and procedures necessary to adequately address the specified task. No evidence is present to suggest that the student has the ability to solve problems of this general type.

## Rubric for Scoring of Mathematical Assessments

Item	Insufficient	Fair	Proficient	Exemplary
Understand Mathematical Concepts and Processes	There are significant omissions or anomalies that indicate a basic lack of comprehension in regard to the mathematical ideas necessary to adequately address the specified task.	The answer contains some of the attributes of an appropriate response. There is some evidence that the student comprehends the essential mathematical ideas addressed by the problem.	There is a mathematically sound approach. There is significant evidence of understanding and errors that may be present do not affect comprehension.	The selected strategy is based on sound conceptual understanding and is successfully implemented.
	Information is either inaccurate or irrelevant.	Some of the relevant information is used.	Most of the relevant information is used.	All of the relevant information is used.
	Math terminology is incorrect.	Most math terminology is used correctly.	Math terminology is used correctly.	Math terminology is used correctly and precisely.
	Unable to recognize patterns and relationships.	Recognizes some patterns and relationships.	Recognized important patterns and relationships.	Creates a general rule or formula that describes the patterns or relationships.
Use of Computations and Procedures	Errors in computations are serious enough to flaw solution.	There is evidence of rationality and purpose in the computation although there may be some computational errors. Inefficient choice procedures impeded success but did not prevent finding a reasonable solution.	Computations were essentially accurate but may contain a minor calculation error that does not alter the accuracy of the answer.	All aspects of the solution are completely accurate. May use multiple ways to compute answer.
	There is no evidence of how the solution was found.	Evidence for the solution is present but may be inconsistent or unclear.	Work clearly supports the solution.	Work clearly supports a thoughtful solution and a rationale is provided that includes criteria (i.e., efficiency, creativity, etc.) for the final choice that was made.
Communicates Mathematical Thinking and Reasoning	Explanation is either not present or unsound.	Explanation may be vague but is understandable.	There is a clear explanation of the work.	Explanation is clear, concise, and logical.
	Mathematical representations did not help clarify thinking.	Mathematical representations are somewhat helpful in clarifying thinking.	Mathematical representations helped clarify the solution.	Mathematical representations clarified the solution and were thorough and complete.