NEW MILFORD PUBLIC SCHOOLS
New Milford, Connecticut

ALGEBRA III

June 2011

Approved by the Board of Education
November 8, 2011

New Milford Board of Education

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New Milford's Mission Statement

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.

Algebra III

After a review of the more challenging topics of second year algebra (quadratic functions, quadratic equations, equations of lines, rational expression simplification, logarithms, etc.), this course will cover topics in polynomial functions, rational functions, systems of equations, complex numbers, and trigonometry. A graphing calculator (TI 83+, TI 84+) is required for this course and is used extensively throughout the year. Students may register for three college credits from Naugatuck Valley Community College.

Common Core State Standards for Mathematics
Mathematics Standards for High School

Standards Key

Number and Quantity

N-RN	The Real Number System
N-Q	Quantities
N-CN	The Complex Number System
N-VM	Vector and Matrix Quantities

Algebra

A-SSE	Seeing Structure in Expressions
A-APR	Arithmetic with Polynomials and Rational Expressions
A-CED	Creating Equations
A-REI	Reasoning with Equations and Inequalities

Functions

F-IF	Interpreting Functions
F-BF	Building Functions
F-LE	Linear, Quadratic, and Exponential Models
F-TF	Trigonometric Functions

Geometry

G-CO	Congruence
G-SRT	Similarity, Right Triangles, and Trigonometry
G-C	Circles
G-GPE	Expressing Geometric Properties with Equations
G-GMD	Geometric Measurement and Dimension
G-MG	Modeling with Geometry

Statistics and Probability

S-ID	Interpreting Categorical and Quantitative Data
S-IC	Making Inferences and Justifying Conclusions
S-CP	Conditional Probability and the Rules of Probability
S-MD	Using Probability to Make Decisions

Pacing Guide

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 1: Fundamental Concepts of Algebra Part 1: Preliminary Review	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 5-6

Identify Desired Results

Common Core State Standards

CC 9-12 N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
CC 9-12 A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.
CC 9-12 A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
CC 9-12 N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.
CC 9-12 A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.
CC 9-12 A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
CC 9-12 A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
CC 9-12 A-REI.2 Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.
CC 9-12 A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
CC 9-12 A-REI.4 Solve quadratic equations in one variable.
CC 9-12 A-REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Values, expressions, and polynomials can be simplified using a specific process. • Exponents and radicals are related to the operation of multiplication. • Polynomials can be added, subtracted, and multiplied to make a more simplified expression.	• How are the properties of exponents related to the basic arithmetic operations? • How do radicals relate to exponents? • How are polynomial expressions combined using operations of addition, subtraction, and multiplication?

• Polynomials can be broken up into products of more simplified terms by factoring. • Rational expressions can be simplified, added, subtracted, multiplied, and divided using the same concepts as with operations on fractions. • Some problems have values that must be excluded from being possible solutions. • Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. • Literal equations can represent many real world situations. • Absolute value represents a positive distance from zero on a number line and may result in two solutions. • Quadratic equations can be solved using a variety of methods, specifically factoring, the quadratic formula, completing the square, and the square root method.	• Why do some expressions such as radical and rational expressions have restrictions under the set of real numbers? • How can rational expressions be simplified? • How are algebraic operations and notation used to simplify and solve equations and inequalities? • How do literal equations apply to real world situations? • How can quadratic equations be solved? • How do absolute value inequalities relate to real world situations?

Expected Performances
What students should know and be able to do

Students will know the following:

- Properties of exponents
- Definition of a radical and of a simplified radical
- Degree of a polynomial
- Addition, subtraction, and multiplication processes of polynomials
- Steps and process to factoring polynomials
- Restrictions on the domain of a rational expressions
- Processes to simplify and perform operations with rational expressions
- Identify and know how to solve linear, rational, literal, and absolute value equations
- The methods and processes to solving a quadratic equation
- The representations and method to solving absolute value inequalities
- Restrictions on the domain of a rational expressions
- Processes to simplify and perform operations with rational expressions
- The methods and processes to solving a quadratic equation
- The representations and method to solving absolute value equations and inequalities

- Key terms: exponents, radicals, rationalize, polynomial, degree, factor, denominator, least common denominator, complex fraction, literal equation, absolute value, complete the square, quadratic formula

Students will be able to do the following:

- Simplify expressions using the properties of exponents
- Simplify radical expressions
- Add, subtract, multiply, and divide radical expressions
- Identify the degree of a polynomial
- Add, subtract, and multiply polynomials
- Factor polynomial expressions
- Identify values that are restricted from the domain of a rational expression
- Simplify, add, subtract, multiply, and divide rational expressions
- Simplify complex fractions
- Identify and know how to solve linear, rational, and literal equations
- Solve quadratic equations using the square root method, factoring, quadratic formula, and by completing the square
- Solve absolute value inequalities
- Solve real world and verbal problems using methods listed above

Character Attributes

- Cooperation
- Honesty
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competency

- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher checks for prerequisite and prior knowledge via warm-up and questioning activities, such as basic review problems on exponents, radicals, polynomial operations, factoring, and solving equations. As the lessons progress, students can also be given questions such as "Find the mistakes in simplifying a rational expression," "Explain how to solve a problem using a specific method (such as completing the square)." Additionally, warm up should contain review on previous material covered during the unit to ensure retention and mastery.
- Teacher introduces the properties of exponents as "short cuts" to what exponents mean (e.g., $x^3 \bullet x^2 = x^5$ since $x^3 = xxx$, $x^2 = xx$, giving a "string" of 5 x's). As properties are introduced, more complex problems will be given to simplify as a class.

- Teacher reviews radicals and models how to simplify them. As lesson progresses from simplifying single radicals to multiplying, adding, and subtracting, to rationalizing, to fractional exponents, teacher leads class through each process, having students share ideas and thoughts on the process.
- Teacher models factoring of a polynomial expression beginning with GCF and grouping one day, then difference of squares and sum and difference of cubes the next day, then trinomials. Teacher allows students many opportunities for independent practice and teacher created groups throughout this topic.
- Connections are made to the similarities in the process used with rational expressions versus basic fractions. Teacher explains the specific steps to the process of simplifying rational expressions. Teacher allows much time for practice with each operation and reviews the differences in the steps required for each type of problem. Supplemental worksheets and board problems should be used to assess mastery of this process.
- Teacher reviews the concept of absolute value as it relates to the number line. Distance interpretation is used as an option to solving absolute value equations and inequalities.
- Teacher discusses the need for alternative methods for solving quadratic equations. Modeling of examples that do not factor should be used to show the methods of completing the square and the quadratic formula. Ample time and practice of each method are given.
- Teacher uses song and/or a "story" as a way to memorize the quadratic formula.

Suggested Learning Activities:

- Students will work independently and as a class simplifying expressions with exponents.
- Students will simplify radicals by working in teacher created groups and/or independently.
- Students will use the white boards to practice factoring polynomial expressions.
- Students will verbally explain the process to add and subtract rational expressions.
- Students will work independently and in small groups to practice simplifying rational expressions.
- Students will practice solving linear equations, literal equations, absolute value equations, and inequalities as a class and independently.
- Students will solve quadratic equations using the methods of factoring, quadratic formula, completing the square, and the square root method by working in pairs.
- Students will verbally state the quadratic formula from memory and may use song or a story as a way of stating it.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: To identify correct and incorrect steps to solving an algebraic equation **Role**: Teacher **Audience**: Student who solved the problem **Situation**: Students are given a problem with specific steps shown as a solution. Students then identify if each step is correct or incorrect and explain why. **Product**: Corrected problem to include feedback and explanation **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work through board work, group work, questioning, and walk-arounds • Check for understanding via going over homework, board and white board activities, and medium such as reflections and exit tickets • Quizzes • Test (may include 10-20 multiple choice, 15-30 regular answer)

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- Graphing calculator
- Supplemental worksheets

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 2: Graphs, Functions, and Models	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 4

Identify Desired Results

Common Core State Standards

- **CC 9-12 A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane often forming a curve (which could be a line).
- **CC 9-12 F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
- **CC 9-12 F-IF.2** Use function notation to evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.
- **CC 9 12 F-1F.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior, and periodicity.
- **CC 9-12 F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **CC 9-12 F-IF.7**: Graph functions expressed symbolically and show key features of the graph by hand, in simple cases, and using technology for more complicated cases.
- **CC 9-12 F-IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **CC 9-12 F-BF.1c (+)** Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- **CC 9-12 F-BF.4** Find inverse functions.
- **CC 9-12 F-BF.4b (+)** Verify by composition that one function is the inverse of another.
- **CC 9-12 G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Linear, quadratic, absolute value, and cubic functions have different shapes when graphed. • Real world situations can often be represented graphically. • Intercepts have special meanings in graphs. • Linear equations can be represented in many forms. • The slope of a line represents the rate of change for a specific relation. • The distance formula and equation of a circle are derived from the Pythagorean Theorem. • Functions are special relations and have a domain and a range. • Some functions have restrictions on their domain. • Graphs of functions can be used to find the domain, range, and intercepts and to tell the nature of the function (increasing, decreasing, constant).	• How does the relationship between variables help us to understand real world situations? • How does visualizing values as points on a graph help us? • What are the relationships between equations of functions and their graphs? • What types of real world problems can be modeled with a linear equation? • Why is slope useful? • Why is it important to understand a circle as both a geometric shape and an algebraic equation? • Why do some functions have restricted values?

Expected Performances
What students should know and be able to do

Students will know the following:

- The formula for slope
- Equations of lines
- Distance and midpoint formulas
- The equation of a circle
- What makes a function
- Domain, range, and function notation
- What the graph of linear, absolute value, quadratic, cubic, and square root functions look like
- Composition of functions
- Inverse of a function
- Key Terms: Intercept, line, slope, slope-intercept form, point-slope form, distance, midpoint, circle, center, radius, function, domain, range, relation, function notation, relative maxima and minima, increasing function, decreasing function, constant function, inverse function, vertical line test, horizontal line test

Students will be able to do the following:

- Graph various equations using different methods including making a T-table and by using the graphing calculator
- Find the slope of a line given two points or an equation
- Graph a line using slope-intercept form
- Write an equation of a line in slope-intercept, point-slope, and undefined slope form
- Find the distance and midpoint of a segment given the endpoints
- Put the equation of a circle in standard form in order to identify the center and radius, graph a circle from the equation, and write an equation given the graph
- Identify whether a graph or equation is a function and find the domain and range of a function
- Identify intercepts, points of relative maxima and minima, intervals where the function is increasing, decreasing, or constant, as well as find specific values from the graph of a function
- Perform operations involving the composition of functions
- Find the inverse of a function

Character Attributes

- Cooperation
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competencies

- Students show graphic representation of data.
- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher gives a quick review of graphing on the rectangular coordinate system, labeling points. Teacher models graphing equations using a T-table (graphic organizer) and allows students class time to practice graphing various equations.
- Teacher uses the overhead graphing calculator to demonstrate how to graph equations. Teacher discusses how to change the viewing window and identify critical values such as intercepts and maximum and minimum values. Modeling of application problems and interpreting data are facilitated by the teacher.
- Teacher gives a warm-up question about slope of a line. The concept of slope and different types of slope are discussed. Slope formula is written on the board and is modeled. Teacher reviews the slope-intercept equation of a line and gives students practice finding slope and graphing lines using the slope-intercept form.
- Teacher reviews all equations of lines and models examples on how to write equations given a variety of information. Class work is given for independent practice.

- Teacher reviews the Pythagorean Theorem as a way to lead into the derivation of the distance formula. Teacher also reviews the midpoint formula. Both formulas are written on the board, and problems are modeled by the teacher.
- Teacher relates the equation of a circle to the Pythagorean Theorem. The standard form is given and teacher models how to complete the square and how to find the center and radius and graph a circle.
- Teacher gives a warm-up on functions to assess what students remember. Relation, function, domain, range, and function notation are defined and identified from examples. Teacher leads class through problems on evaluating functions for specific values.
- Teacher reviews how to graph functions and then facilitates a discussion on how to find the domain, range, intercepts, and values from the graph. Teacher models how to identify increasing, decreasing, and constant values using interval notation.
- Teacher discusses the origins of the infinity symbol as it relates to the mobius strip. Teacher leads class through hands-on activities pertaining to the mobius strip and its properties.
- Teacher models the process to find the composition of functions and inverse of a function. Teacher gives problems for students to complete at their seats or at the board.

Suggested Learning Activities:

- Students will work independently graphing different equations by making a T-table.
- Students will practice using the graphing calculator to graph equations, change the viewing window, and find intervals and other values using various tools such as trace and table. Students will work in pairs graphing equations as well as reading and interpreting real-world graphs.
- Students will calculate the slope of a line and graph lines using slope intercept form by working in teacher created small groups or independently.
- Students will write equations of lines given different situations.
- Students will work in teacher created groups to find the distance between two points and the midpoint of a segment.
- Students will find the center and radius of a circle and complete the square to get the equation in graphing form. Students will put answers on the board for other students to compare and discuss.
- Students will identify functions and find their domain and range. Students will also evaluate functions.
- Students will graph functions and find the domain, range, and other critical values from graphs of functions. Students will complete a review worksheet on functions and will put their answers on the board.
- Students will view a video and then complete an activity on the mobius strip.
- Students will work independently and in pairs to find the composition of functions and the inverse of functions.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: To interpret real-world graphs, write equations from the graphs, and make predictions from that equation **Role**: Statistician **Audience**: Managers of two companies **Situation**: Students are given two graphs – one dealing with the average cost of a retail prescription and the other dealing with the number of workers per Social Security beneficiary. Students are to identify and interpret the y-intercept, slope, linear equation, and explain the meanings of these values. **Product**: Students will write future predictions based on their findings. **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work through board work, group work, questioning, and walk-arounds • Quizzes • Test on unit two (may include 3-5 multiple choice, 25 regular answer, one essay) • Review quiz on material covered in units one and two

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- "No Magic at All: Mobius Strip." *Metacafe*. Web. 14 June 2011.
- Worksheet supplements
- Graphing calculator

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 3: Polynomial and Rational Functions	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 3

Identify Desired Results

Common Core State Standards

- **CC 9-12 N-CN.1** Know there is a complex number *i* such that $i^2 = \sqrt{-1}$ and every complex number has the form *a* + *bi* with *a* and *b* real.
- **CC 9-12 N-CN.2:** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- **CC 9-12 N-CN.3 (+)** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **CC 9-12 A-SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **CC 9-12 A-SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **CC 9-12 A-APR.2** Know and apply the Remainder Theorem: for a polynomial *p*(*x*) and a number a, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*).
- **CC 9-12 A-APR.3** Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
- **CC 9-12 A-APR.6** Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*) where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **CC 9-12 F-IF.7**: Graph functions expressed symbolically and show key features of the graph by hand, in simple cases, and using technology for more complicated cases.
- **CC 9-12 F-IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **CC 9-12 F-IF.7d (+)** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- **CC 9-12 F-IF.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• The value $\sqrt{-1}$ can be represented as an imaginary number (i).	• Why are some values not considered real numbers? • Where did complex numbers originate, and how do they fit into the algebraic framework?

• Complex numbers combine real and imaginary numbers and can have operations of addition, subtraction, multiplication, and division performed on them. • Quadratic functions are shaped like parabolas and have special properties. • Polynomial functions can be graphed to find zeros and maximum and minimum values • Long and synthetic division can be used to find the zeros of a polynomial. • The remainder and factor theorems help find specific values of a polynomial and are used to check if a value is a zero. • Rational functions have special behaviors when graphed and have undefined values in their domain and range, leading to asymptotic behavior.	• How do quadratic functions relate to real world situations? • How can technology be used to represent functions and to verify solutions found manually? • What is the importance of finding values such as intercepts and maximum/minimum from a graph? • Why do different functions have different graphs and behaviors?

Expected Performances
What students should know and be able to do

Students will know the following:

- Definition of imaginary and complex numbers
- Quadratic functions – definition, equation, graphing form, and how to graph
- Polynomial functions and their graphs
- Long and synthetic division
- Remainder and Factor Theorems
- Rational functions, asymptotes
- Key Terms: imaginary number, complex number, quadratic function, parabola, vertex, axis of symmetry, x- and y- intercepts, domain, range, polynomial function, zeros, roots, long division, synthetic division, Remainder Theorem, Factor Theorem, rational function, asymptote

Students will be able to do the following:

- Simplify imaginary and complex numbers and perform mathematical operations with them
- Graph quadratic functions and identify the vertex, axis of symmetry, direction of opening, maximum or minimum value, x- and y- intercepts, domain, and range.
- Complete the square to get a quadratic function in graphing form
- Graph polynomial functions by finding the zeros and identifying the nature of the curve
- Use long and synthetic division to divide polynomials and find the remainder

- Apply the Remainder and Factor Theorems to find the zeros of a polynomial or to tell if a value is a zero
- Graph a rational function by finding the vertical, horizontal, and/or slant asymptotes, and the x- and y- intercepts

Character Attributes

- Cooperation
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competencies

- Students show graphic representation of data.
- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher gives warm-up questions to lead into and review the concept of an imaginary number. Complex numbers are also discussed, and teacher models examples of simplifying and performing operations of addition, subtraction, multiplication, and division with complex numbers.
- Teacher uses the overhead graphing calculator or Smart Board technology to facilitate class discovery of the effect the values a, h, and k have on the graphing form of a quadratic function ($y = a(x-h)^2+k$). Teacher then uses that discovery to introduce the key features of the graph of a parabola: vertex, axis of symmetry, direction of opening, max/min value, x- and y- intercept, domain, and range. Teacher models, with help from students, how to graph a parabola from that information. After mastery of this, teacher builds on the topic by modeling how to get any quadratic function into graphing form by the method of completing the square.
- Teacher introduces the graphs of polynomial functions by using the graphing calculator or Smart Board technology and student discovery of the nature of the graphs. Teacher then models the steps to graphing polynomial functions by hand.
- Teacher reviews long division with numbers to model long division with polynomials. Synthetic division is also modeled. Teacher makes a connection to division of polynomials and whether or not a polynomial is a factor. The Remainder and Factor Theorems are introduced, and the teacher leads students through the process of determining the remainder of the division of two polynomials and whether a polynomial is a factor.
- Teacher gives a warm-up question that reviews domain of a rational function. This leads into the concept of asymptotes for the graph of the rational function. Teacher models the key components of vertical, horizontal, and slant asymptotes and how to use intercepts to determine the general shape of the graph.

Suggested Learning Activities:

- Students will give ideas and examples of imaginary and complex numbers.

- Students will work as a class and then independently simplify and perform mathematical operations with complex numbers. Individual students will put up answers to practice problems on the board.
- Students will discover the properties of the graphing form of a quadratic function by observing the changes of the graph of a parabola when different values are inserted. Students will then work as a class and in pairs to find the key values of a parabola and to graph it. Students' work will be put on the board as a way to review and monitor progress.
- Students will practice completing the square to get a quadratic function in graphing form by working in teacher created groups.
- Students will work independently or in pairs to discover the nature of the graphs of a polynomial function. Students will then practice graphing them by hand. Students will use the think-pair-share method to compare their answers.
- Students will work as a class or in pairs to practice long and synthetic division and to use that division to determine the remainder. Students will apply the Factor Theorem to determine if a polynomial is a factor of a larger degreed polynomial. Student work will be discussed orally as a group.
- Students will be placed in teacher created groups to graph rational functions. After students have mastered one or two, students will work independently on white boards or at the chalk board graphing other rational functions.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: To identify correct and incorrect steps to solving an algebraic equation **Role**: Teacher **Audience**: Student who solved the problem **Situation**: Students are given a problem with specific steps shown as a solution. Students then identify if each step is correct or incorrect and explain why. **Product**: Corrected problem to include feedback and explanation **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work through board work, group work, questioning, and walk-arounds • Quizzes • Test (approximately 5-10 multiple choice, 10-15 regular answer, one explain) • Review quiz on material covered in units one and two

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- Worksheet supplements
- Graphing calculator

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 4: Exponential and Logarithmic Functions	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 3

Identify Desired Results

Common Core State Standards

- **CC 9-12 F-IF.7e** Graph exponential and logarithmic functions showing intercepts, end behavior, trigonometric functions, period, midline, and amplitude.
- **CC 9-12 F-BF.1** Write a function that describes a relationship between two quantities.
- **CC 9-12 F-BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Exponential equations can be solved by getting a common base or by using logarithms. • Logarithms are used to represent exponents, which could not be solved. • The properties of logarithms relate to the properties of exponents.	• What is the value of an exponential equation in the real world? • What does a logarithm represent? • How does the relationship between exponential and logarithmic functions help us?

Expected Performances
What students should know and be able to do

Students will know the following:

- The process to solving exponential equations by getting a common base and by using logarithms
- A logarithm is a way to represent exponents
- The properties of logarithms
- Common logarithm
- Natural logarithm
- Key Terms: exponent, exponential equation, base, logarithm, common logarithm, e, natural logarithm

Students will be able to do the following:

- Change expressions to have the same base in order to solve exponential equations
- Change expressions from exponential form to logarithmic form and vice-versa
- Evaluate logarithmic expressions
- Apply the properties of logarithm to solve logarithmic equations

- Use common logarithms to solve exponential equations that do not have a common base
- Identify a natural logarithm as log base

Character Attributes

- Cooperation
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competency

- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher uses independent/guided practice via supplemental worksheets to review simplifying expressions with exponents. Teacher walks around and monitors student progress, assists individual students, and models examples when needed for the class.
- Teacher gives a warm-up question on exponents as a way to lead in to solving exponential equations. Teacher models different examples of exponential equations that have the same base and the process to solving them.
- Teacher has the class graph the equation $y = 2^x$ and its inverse as a way of introducing the graph of an exponential equation and a logarithm.
- Teacher models how to solve and evaluate logarithmic equations and expressions by changing to exponential form and by applying the properties of logarithms.

Suggested Learning Activities:

- Students will work independently and in teacher created groups to complete practice problems that review exponents. Students will use think-pair-share to compare and discuss their answers.
- Students will complete problems on solving exponential equations. Students will volunteer their solutions and will explain the process they used.
- Students will practice evaluating and solving logarithmic expressions and equations by various activities such as independent practice, board work, think-pair-share and/or use of white boards.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
	• Monitoring class work through board work, group work, questioning, and walk-arounds

	• Quizzes • Test (approximately 20 regular answer, three explain)

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- Supplement worksheets on exponents and logarithms (may replace or supplement textbook assignments)

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 5: Trigonometric Functions	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 6-7

Identify Desired Results

Common Core State Standards

- **CC 9-12 G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- **CC 9-12 F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **CC 9-12 F-TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **CC 9-12 F-TF.3 (+)** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.
- **CC 9-12 F-TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- **CC 9-12 F-TF.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Right triangle trigonometry has many uses and applications. • The unit circle can represent angles of any measure, in degrees or radians, and is cyclic. • The trigonometric functions of specific angles relate to specific points and values on the unit circle. • The graphs of the trigonometric functions are cyclic with certain traits. Sine and cosine graphs produce “waves.”	• How can the use of right triangles, trigonometric functions, and the Pythagorean Theorem be used to solve real world problems? • What is the unit circle and why is it important in trigonometry? • Why do the graphs of trigonometric functions look the way they do? • How do the graphs of sine and cosine apply to real life applications? • What are the uses of inverse trigonometric functions?

Expected Performances
What students should know and be able to do

Students will know the following:

- Pythagorean Theorem
- Definitions of the six trigonometric functions

- Radian measures
- Angles, points, and trigonometric values on the unit circle
- Reference angles
- Methods to graphing sine, cosine, tangent, cosecant, and secant, and their general behaviors
- Inverse trigonometric functions and the restrictions on their ranges
- Compositions of trigonometric functions
- Key Terms: Pythagorean Theorem, sine, cosine, tangent, cosecant, secant, cotangent, radian, degree, coterminal, complementary, supplementary, unit circle, angle in standard position, cyclic, reference angle, amplitude, period, phase shift, vertical shift, inverse trigonometric functions

Students will be able to do the following:

- Use the Pythagorean Theorem and right triangle trigonometry to solve right triangles
- Define the six trigonometric functions
- Construct a unit circle and identify angles in both degree and radian measures
- Convert degrees to radians (and vice versa)
- Identify specific points on the unit circle
- Define the trigonometric functions as related to the x and y coordinates and radius on the unit circle
- Use reference angles and definitions of the trigonometric functions to find the specific values on the unit circle.
- Fill in the trigonometric table for values of special and quadrantile angles
- Graph sine, cosine, tangent, cosecant, and secant functions and identify special characteristics such as amplitude, period, phase shift, and vertical shift
- Identify the domain and range of inverse trigonometric functions
- Find the exact values of inverse trigonometric functions and composite trigonometric functions
- Use graphing calculators to check graphs of trigonometric values and to find approximate solutions to problems

Character Attributes

- Cooperation
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competencies

- Students show graphic representation of data.
- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher reviews the Pythagorean Theorem and right triangle trigonometry. Teacher gives review and practice problems as class work to find missing sides and angles. Lesson leads into the introduction of the three reciprocal trigonometric functions, and applications of trigonometry will be discussed.
- Teacher introduces the concept of the Unit Circle by first discussing radian measure as a representation of the length of the arc on the circle. Teacher leads class through the discovery of the relationship between degrees and radians and how to convert degrees to radians and radians to degrees.
- Teacher continues to demonstrate the relations on the Unit Circle to points on the circle and angle measures. After a review of special right triangles, teacher models how to find specific points given particular reference angles.
- Teacher prepares materials (garland, laminated color coded cards with degree measures, radian measures, and coordinates of points) for the Unit Circle activity where students physically construct a model of the Unit Circle.
- Teacher reviews the relationships of trigonometric functions in right triangles and then shows the connection with the x, y, and r values of the Unit Circle. Lesson leads into applications of the trigonometric functions to any point in the coordinate plane, which the teacher models and explains.
- With the help of the graphing calculator, teacher leads class through graphing the sine and cosine curves. Discussion on the general shape of the curves, their periodic behavior, and their amplitude, period, phase shift, and vertical shift occurs. Teacher leads class through examples on how to graph sine and cosine functions.
- Teacher has class make a t-table to graph the tangent curve. The general shape of the curve and its period will be discussed. Teacher models how to graph the secant and cosecant functions by using the sine and cosine graphs as "helpers."
- Teacher models how to use the graphing calculator and trigonometric table to find approximated and exact values of inverse trigonometric functions. Teacher also explains how to find composite trigonometric values.
- Teacher determines cooperative groups for various activities during this unit.

Suggested Learning Activities:

- Students will practice solving right triangles using trigonometry by working in teacher created groups. Students will also identify the values of the reciprocal functions and will use their calculators to find specific values.
- Students will work independently on changing measures from degrees to radians and radians to degrees. Students may collaborate with a partner on their solutions.
- Students will complete the Unit Circle chart with specific degree measure, radian measure, and the coordinate of the associated points. Students will then use the Unit Circle and reference angles to fill in the trigonometric table.
- Students will work cooperatively as a group to construct the Unit Circle in the rotunda using garland and laminated values on the circle. Students will then play the "Move It" game where they must move to a specific value on the circle.
- Students will apply the definitions of the trigonometric functions to the Unit Circle. Students will then find the values of the trigonometric functions at any value.

- Students will work at the board to practice graphing sine and cosine functions, identifying the amplitude, period, phase shift, and vertical shift.
- Students will graph y=tanx as well as various cosecant and secant curves by working in teacher created groups.
- Students will work in pairs to find inverse and composite trigonometric functions using their calculators and the trigonometric table.

Assessments

Performance Tasks Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Performance Task 1 **Goal**: To use right triangle trigonometry to solve real-world application problems **Role**: Surveyor **Audience**: Land development company **Situation**: Given various situations, you are to calculate unknown distances to report to the land development company for construction purposes **Product**: Calculated distances with work shown **Standards for Success**: Mathematic department scoring rubric **Performance Task 2** **Goal**: To calculate your personal biorhythm chart for the current month **Role**: Social scientist **Audience**: School staff **Situation**: You are to convince school staff whether or not your personal academic performance will be stronger or weaker based on your biorhythm **Product**: Your completed boirhthym **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work through board work, group work, questioning, and walk-arounds • Quizzes • Test (approximately 25-40 regular answer, approximately five graphs, 20-25 short answer)

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- Supplemental worksheets
- Graphing calculator

New Milford Public Schools

Committee Members: Cheryl Reiner & Deborah Murnan Unit 6: Additional Topics in Trigonometry	Course/Subject: Algebra III Grade Level: 12 # of Weeks: 2 (as time permits)

Identify Desired Results

Common Core State Standards

- **CC 9-12 G-SRT.11 (+)** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
- **CC 9-12 N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• The Law of Sines and Law of Cosines apply to non-right triangles and can be used to find missing lengths or angles.	• How can the use of trigonometric functions be extended to solve word problems and triangles with no right angles?

Expected Performances
What students should know and be able to do

Students will know the following:

- Law of Sines
- Law of Cosines
- Applications of the Law of Sines and Law of Cosines
- Key Terms: Law of Sines, Law of Cosines, oblique triangle, ambiguous case

Students will be able to do the following:

- Solve a triangle for missing sides or angles using the Law of Sines and the Law of Cosines
- Apply the ambiguous of the Law of Sines to determine if there are none, one, or two possible triangles

Character Attributes

- Cooperation
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competency

- Students use technology tools (i.e., calculators, data collection probes, videos, educational software) for problem solving, self-directed learning, and extended learning activities.

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher introduces Law of Sines by having students solve a right triangle. Discussion takes place about solving a non-right triangle, and teacher gives the formula for the Law of Sines and models some examples. Teacher explains the cases when the Law of Sines can be used (AAS, ASA, SSA).
- Teacher goes over the ambiguous case for the Law of Sines and demonstrates why there are possibly no triangles, one triangle, or two triangles. Teacher models examples with no triangle and with two triangles.
- Teacher gives the formula for the Law of Cosines and discusses when to use it (SSS, SAS cases). Teacher models an example where a side should be found first, and then one where an angle should be found first.

Suggested Learning Activities:

- Students will work as a whole group to practice some examples on the Law of Sines (non ambiguous case).
- Students will practice examples of the ambiguous case of the Law of Sines by working in small teacher created groups.
- Students will solve triangles using the Law of Cosines and will compare their answers with a partner.
- If time permits, students will work independently to complete the performance task related to applications with the Law of Sines and Law of Cosines.

Assessments

Performance Task (if time permits) Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: To find unknown values in specific real-world situations **Role**: Surveyor **Audience**: Manager of a development company **Situation**: Given various situations, use the Laws of Sines and Cosines to calculate values that are otherwise non-measurable (e.g., calculate the distance between two landmarks that have a lake between them). **Product**: Calculated distances with solutions shown **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work through board work, group work, questioning, and walk-arounds • Quizzes • Test (approximately five regular answer, one explain, and three application problems)

Suggested Resources

- Textbook: Blitzer, Robert. Precalculus. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2004. Print.
- Supplement worksheets
- Graphing calculator

Final Course Assessments

The midyear assessment consists of approximately twenty-four multiple choice questions and sixty questions that require work to be shown, an explanation, a graph, or a written response.

The final course assessment consists of approximately twenty-four multiple choice questions and thirty questions that require work to be shown, an explanation, a graph, or a written response.

Rubrics are used for judging the success of major projects, papers, and presentations.

Third Generation CAPT Scoring Rubric

Score 3
The student has demonstrated a **full and complete** understanding of all concepts and processes essential to this application. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student's competence in problem-solving and reasoning, computing and estimating, and communicating to the full extent that these processes apply to the specified task. The response may, however, contain minor arithmetic errors that do not detract from a demonstration of full understanding. Student work is shown or an explanation is included.

Score 2
The student has demonstrated a **reasonable** understanding of the essential mathematical concepts and processes in this application. The student's response contains most of the attributes of an appropriate response including a mathematically sound approach and evidence of competence with applicable mathematical processes, but contains flaws that do not diminish the evidence that the student comprehends the essential mathematical ideas addressed in the task. Such flaws include errors attributed to faulty reading, writing, or drawing skills; errors attributed to insufficient, non-mathematical knowledge; and errors attributed to careless execution of mathematical processes or algorithms.

Score 1
The student has demonstrated a **partial** understanding of some of the concepts and processes in this application. The student's response contains some of the attributes of an appropriate response, but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge; errors in fundamental mathematical procedures; and other omissions or irregularities that bring into question the extent of the student's ability to solve problems of this general type.

Score 0
The student has demonstrated **merely an acquaintance** with the topic. The student's response is associated with the task in the item but contains few attributes of an appropriate response. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical ideas and procedures necessary to adequately address the specified task. No evidence is present to suggest that the student has the ability to solve problems of this general type.

Rubric for the Scoring of Mathematics Assessments

Item	Insufficient	Fair	Proficient	Exemplary
Understand Mathematical Concepts and Processes	There are significant omissions or anomalies that indicate a basic lack of comprehension in regard to the mathematical ideas necessary to adequately address the specified task.	The answer contains some of the attributes of an appropriate response. There is some evidence that the student comprehends the essential mathematical ideas addressed by the problem.	There is a mathematically sound approach. There is significant evidence of understanding and errors that may be present do not affect comprehension.	The selected strategy is based on sound conceptual understanding and is successfully implemented.
	Information is either inaccurate or irrelevant.	Some of the relevant information is used.	Most of the relevant information is used.	All of the relevant information is used.
	Math terminology is incorrect.	Most math terminology is used correctly.	Math terminology is used correctly.	Math terminology is used correctly and precisely.
	Unable to recognize patterns and relationships.	Recognizes some patterns and relationships.	Recognized important patterns and relationships.	Creates a general rule or formula that describes the patterns or relationships.
Use of Computations and Procedures	Errors in computation are serious enough to flaw solution.	There is evidence of rationality and purpose in the computation although there may be some computational errors. Inefficient choice procedures impeded success but did not prevent finding a reasonable solution.	Computations were essentially accurate but may contain a minor calculation error that does not alter the accuracy of the answer.	All aspects of solution are completely accurate. May use multiple ways to compute answer.
	There is no evidence of how the solution was found.	Evidence for the solution is present but may be inconsistent or unclear.	Work clearly supports the solution.	Work clearly supports a thoughtful solution and a rationale is provided that includes criteria (i.e., efficiency, creativity, etc.) for the final choice that was made.
Communicates Mathematical Thinking and Reasoning	Explanation is either not present or unsound.	Explanation may be vague but is understandable.	There is a clear explanation of the work.	Explanation is clear, concise, and logical.
	Mathematical representations did not help clarify thinking.	Mathematical representations are somewhat helpful in clarifying thinking.	Mathematical representations helped clarify the solution.	Mathematical representations clarified the solution and were thorough and complete.