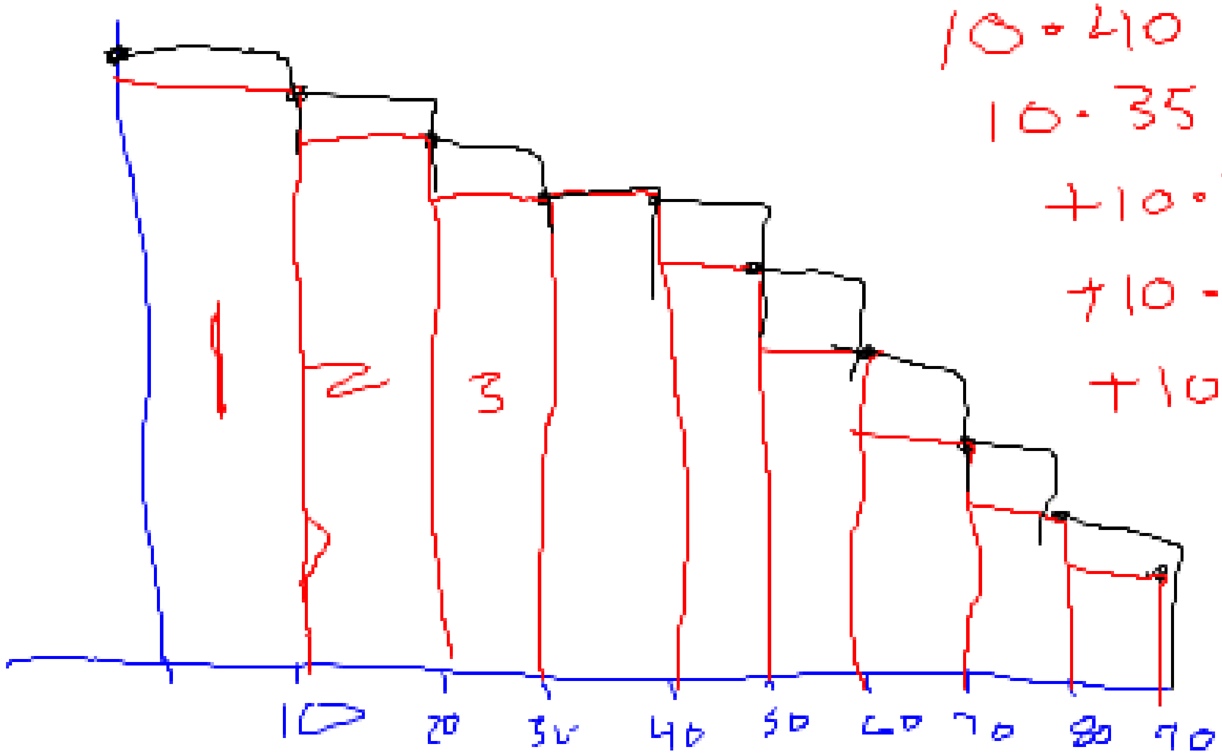
Intro to Riemann Sums

Using rectangles and trapezoids to approximate area under a curve.

Exploration 1: A tank is being filled with water using a pump that is old, and slows down as it runs. The table below gives the rate at which the pump pumps at ten-minute intervals. If the tank is initially empty, how many gallons of water are in the tank after 90 minutes?

Elapsed time (Minutes)	0	10	20	30	40	50	60	70	80	90
Rate (gallons / minute)	42	40	38	35	35	32	28	20	19	10



$10 \cdot 40 + 10 \cdot 38 + 10 \cdot 35 + 10 \cdot 35 + 10 \cdot 32 + 10 \cdot 28 + 10 \cdot 20 + 10 \cdot 19 + 10 \cdot 10$

RHS = 2570 gallons

LHS = 2890 gallons

Exploration 2: The speed of an airplane in miles per hour is given at half-hour intervals in the table below. How far does the airplane travel in the three hours given in the table?

Elapsed time ~~(minutes)~~ hr	0	~~30~~ .5	~~60~~ 1	~~90~~ 1.5	~~120~~ 2	~~150~~ 2.5	~~180~~ 3
Speed (miles per hour)	375	390	400	390	385	350	345

$$LHS = .5\cdot 375 + .5\cdot 390 + .5\cdot 400$$
$$+ .5\cdot 390 + .5\cdot 385$$
$$+ .5\cdot 350$$
$$= 1145 \text{ miles}$$

Extra

sec	0	1	4	10	13	20
cm/sec	5	8	7	6	3	1

$$LHS = 1\cdot\underline{5} + 3\cdot\underline{8} + 6\cdot\underline{7} + 3\cdot\underline{6} + 7\cdot\underline{3}$$

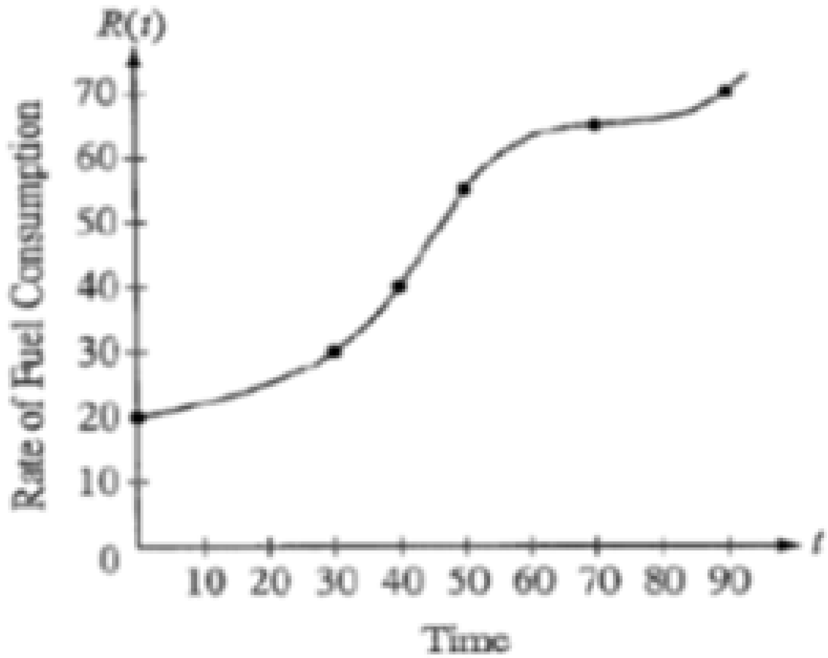
$$= 110\,\text{cm}$$

$$RHS = 1\cdot\underline{8} + 3\cdot\underline{7} + 6\cdot\underline{6} + 3\cdot\underline{3} + 7\cdot\underline{1}$$

$$= 81\,\text{cm}$$

$$\text{Trap. Sum} = \frac{1}{2}\left[1(13) + 3(15) + 6(13) + 3(9) + 7(4)\right]$$

$$= 95.5\,\text{cm}$$



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

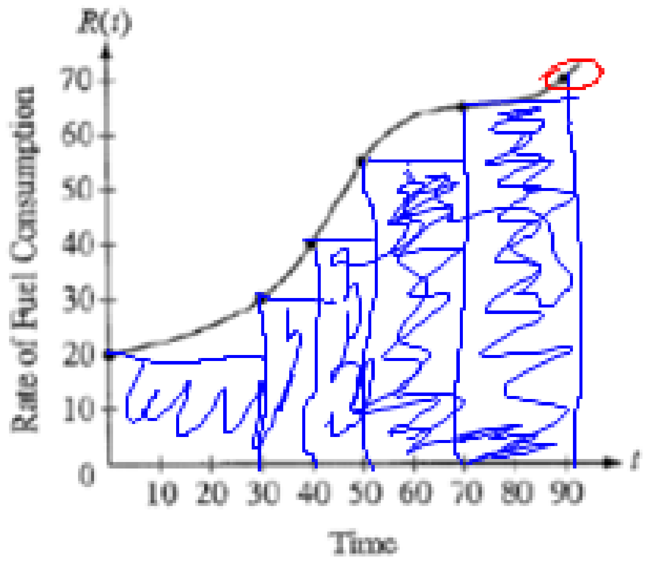
The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of R(t), for the time interval 0 ≤ t ≤ 90 minutes are shown above.

a. Use the data from the table to approximate R'(45). Show the computations that lead to your answer. Indicate units of measure.

slope = $\frac{55-40}{50-40} = 3/2 = 1.5$ gal/min²

b. The rate of fuel consumed is increasing fastest at t = 45 minutes. What is the value of R"(45). Explain your reasoning.

R''(45) = 0 because R'(t) is a max

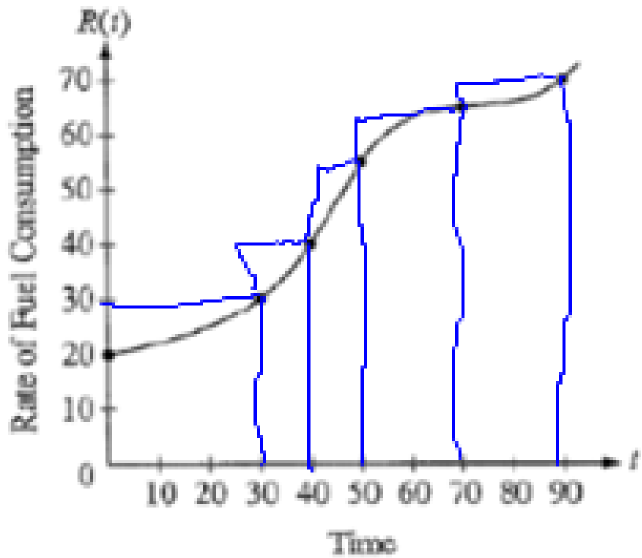


c. Approximate the total fuel consumption by finding the area under the curve. Do so using a Left Riemann Sum with the five subintervals indicated by the data in the table. How does this numerical approximation compare with the total fuel consumption?

$30(20)+10(30)+10(40)+20(55)+20(65)$

3700 gallons

Less than actual

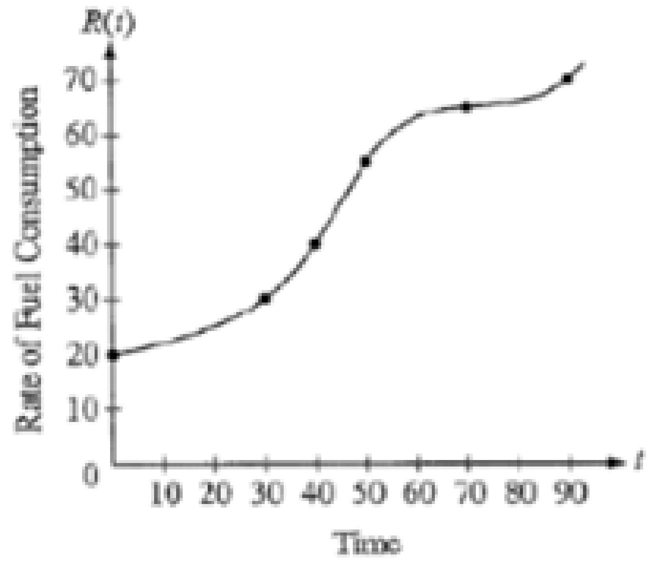


d. Approximate the total fuel consumption by finding the area under the curve. Do so using a Right Riemann Sum with the five subintervals indicated by the data in the table. How does this numerical approximation compare with the total fuel consumption?

$30(30) + 10(40) + 10(55) + 20(65) + 20(70)$

4550 gallons

More than actual



e. Approximate the total fuel consumption by finding the area under the curve. Do so using a Trapezoidal Riemann Sum with the five subintervals indicated by the data in the table. How does this numerical approximation compare with the answers found using Left and Right Riemann Sums?

f. What are some possibilities to ensure that we account for ALL of the area under the curve?

2008 AB 2 BC 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 p.m. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.

2011 AB 2 BC 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and the temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

   (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

   (c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

   (b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

   (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

   (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

2007 AB #5