DeSoto
COUNTY SCHOOLS

Geometry

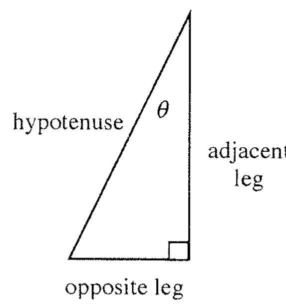
Week 4

MORE TRIGONOMETRY 5.1.1 – 5.1.3

We next introduce two more trigonometric ratios: sine and cosine. Both of them are used with acute angles of right triangles, just as the tangent ratio is. Using the diagram below:

$\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$ $\qquad\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$

and from Chapter 4: $\qquad\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$



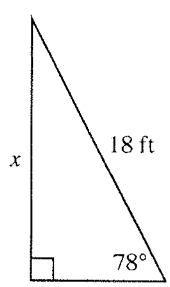
Note: If you decide to use the other acute angle in the triangle, then the names of the legs switch places. The opposite leg is always across the triangle from the acute angle you are using.

See the Math Notes boxes in Lessons 5.1.2 and 5.1.4.

Example 1

Use the sine ratio to find the length of the unknown side in each triangle below.

a.



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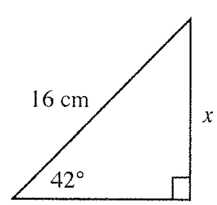


The sine of the angle is the ratio $\frac{\text{opposite leg}}{\text{hypotenuse}}$. For part (a) we will use the 78° as θ. From the 78° angle, we find which side of the triangle is the opposite leg and which side is the hypotenuse. The hypotenuse is always the longest side, and it is always opposite the right angle. In this case, it is 18. From the 78° angle, the opposite leg is the side labeled x. Now we can write the equation at right and solve it.

$$\sin 78° = \tfrac{x}{18} \quad \left(\tfrac{\text{opposite}}{\text{hypotenuse}}\right)$$
$$18\sin 78° = x$$
$$x \approx 17.61 \text{ ft}$$

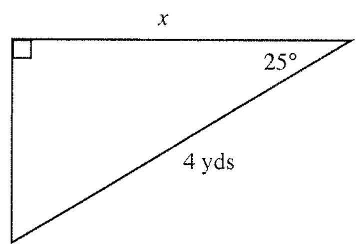
In part (b), from the 42° angle, the opposite leg is x and the hypotenuse is 16. We can write and solve the equation at right. Note: In most cases, it is most efficient to wait until the equation has been solved for x, then use your calculator to combine the values, as shown in these examples.

$$\sin 42° = \tfrac{x}{16}$$
$$16(\sin 42°) = x$$
$$x \approx 10.71 \text{ cm}$$

Example 2

Use the cosine ratio to find the length of the unknown side in each triangle below.

a.



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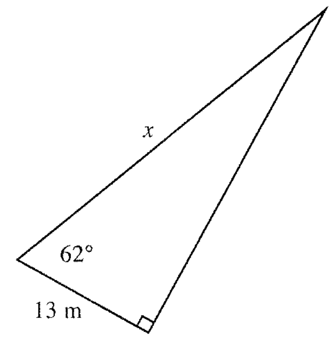


Just as before, we set up an equation using the cosine ratio, $\frac{\text{adjacent leg}}{\text{hypotenuse}}$. Remember that you can always rotate the page, or trace and rotate the triangle, if the figure's orientation is causing confusion. The key to solving these problems is recognizing which side is adjacent, which is opposite, and which is the hypotenuse. (See the box above Example 1 for this information.) For part (a), the angle is 25°, so we can write and solve the equation at right.

$$\cos 25° = \frac{x}{4} \quad \left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$$
$$4(\cos 25°) = x$$
$$x \approx 3.63 \text{ yds}$$

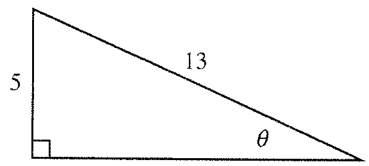
In part (b), from the 62° angle, the adjacent leg is 13 and the hypotenuse is x. This time, our variable will be in the denominator. As we saw in earlier chapters, this will add one more step to the solution.

$$\cos 62° = \frac{13}{x}$$
$$x \cos 62° = 13$$
$$x = \frac{13}{\cos 62°} \approx 27.69 \text{ m}$$

Example 3

In each triangle below, use the inverse trigonometry buttons on your calculator to find the measure of the angle θ to the nearest hundredth.

a.



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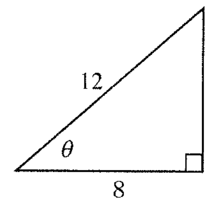
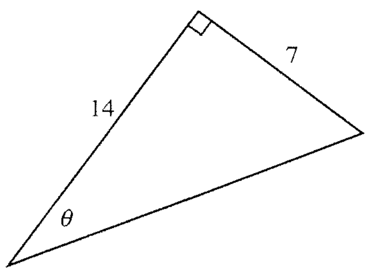


c.



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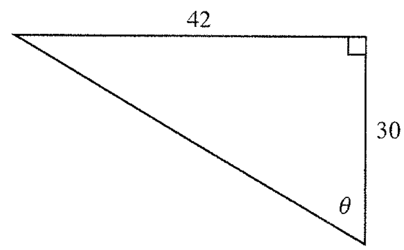


For each of these problems you must decide whether you will be using sine, cosine, or tangent to find the value of θ. In part (a), if we are standing at the angle θ, then 5 is the length of the opposite leg and 13 is the length of the hypotenuse. This tells us to use the sine ratio. For the best accuracy, enter the ratio, not its decimal approximation.

$$\sin\theta = \tfrac{5}{13}$$
$$\sin\theta \approx 0.385$$

To find the value of θ, find the button on the calculator that says $\sin^{-1}$. (Note: Calculator sequences shown are for most graphing calculators. Some calculators use a different order of keystrokes.) This is the "inverse sine" key, and when a ratio is entered, this button tells you the measure of the angle that has that sine ratio. Here we find $\sin^{-1}\tfrac{5}{13} \approx 22.62°$ by entering "2nd," "sin," (5 ÷ 13), "enter." Be sure to use parentheses as shown.

In part (b), 8 is the length of the adjacent leg and 12 is the length of the hypotenuse. This combination of sides fits the cosine ratio. We use the $\cos^{-1}$ button to find the measure of θ by entering the following sequence on the calculator: "2nd," "cos," (8 ÷ 12), "enter."

$$\cos\theta = \tfrac{8}{12}$$
$$\cos\theta \approx 0.667$$
$$\theta = \cos^{-1}\tfrac{8}{12}$$
$$\theta \approx 48.19°$$

In part (c), from θ, 7 is the length of the opposite leg and 14 is the length of the adjacent leg. These two sides fit the tangent ratio. As before, you need to find the $\tan^{-1}$ button on the calculator.

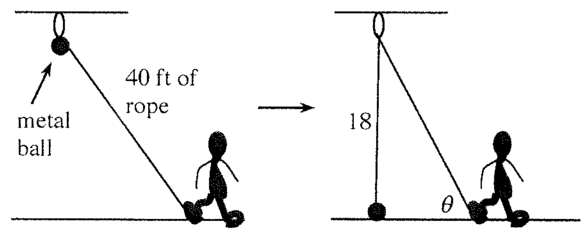
$$\tan\theta = \tfrac{7}{14} = 0.5$$
$$\tan\theta = 0.5$$
$$\theta = \tan^{-1} 0.5 \approx 26.57°$$

If we are standing at the angle θ in part (d), 42 is the length of the opposite leg while 30 is the length of the adjacent leg. We will use the tangent ratio to find the value of θ.

$$\tan\theta = \tfrac{42}{30} = 1.4$$
$$\tan\theta = 1.4$$
$$\theta = \tan^{-1} 1.4 \approx 54.46°$$

Example 4

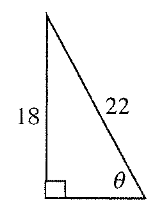
Kennedy is standing on the end of a rope that is 40 feet long and threaded through a pulley. The rope is holding a large metal ball 18 feet above the floor. Kennedy slowly slides her feet closer to the pulley to lower the ball. When the ball hits the floor, what angle (θ) does the rope make with the floor where it is under her foot?



As always, we must draw a picture of this situation to determine what we must do. We start with a picture of the beginning situation, before Kennedy has started lowering the ball. The second picture shows the situation once the ball has reached the floor. We want to find the angle θ. You should see a right triangle emerging, made of the rope and the floor. The 40-foot rope makes up two sides of the triangle: 18 feet is the length of the leg opposite θ, and the rest of the rope, 22 feet of it, is the hypotenuse. With this information, draw one more picture. This one will show the simple triangle that represents this situation.

From θ, we have the lengths of the opposite leg and the hypotenuse. This tells us to use the sine ratio.

$$\sin\theta = \tfrac{18}{22}$$
$$\theta = \sin^{-1}\tfrac{18}{22}$$
$$\theta \approx 54.9°$$



Problems

Using the tangent, sine, and cosine buttons on your calculator, calculate the value of x to the nearest hundredth.

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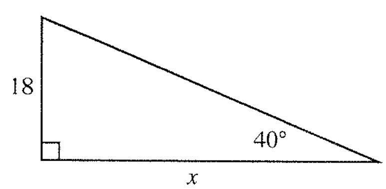

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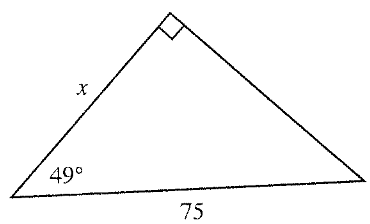

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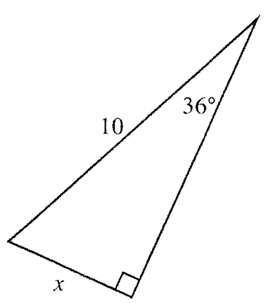

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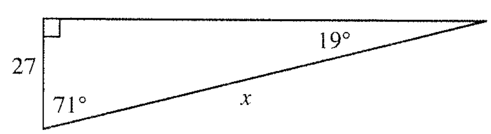
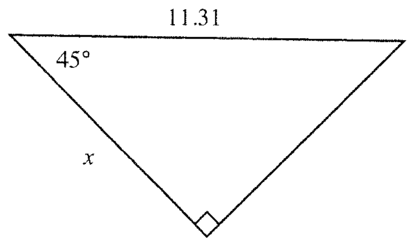
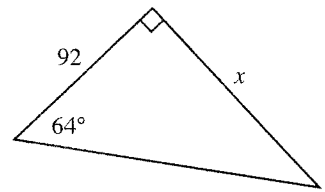
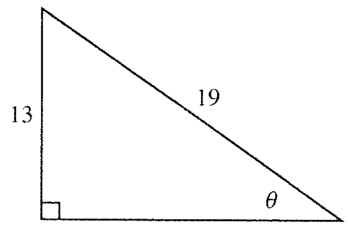
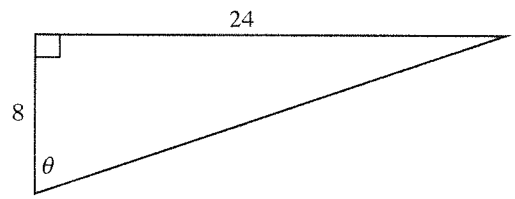


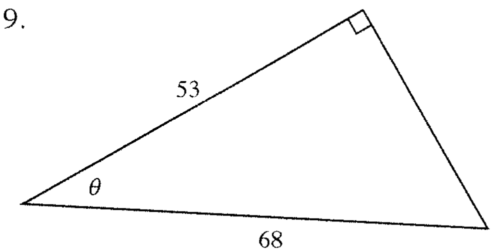
5. 

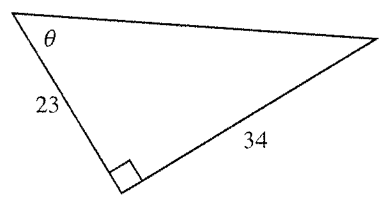
6. 

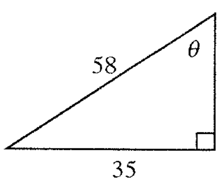
Using the $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ buttons on your calculator, calculate the value of θ to the nearest hundredth.

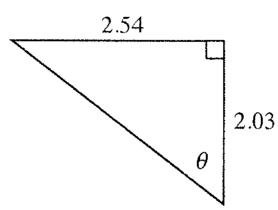
7. 

8. 

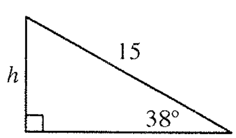
9. 

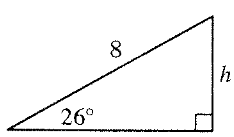
10. 

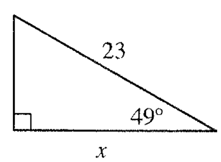
11. 

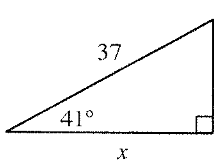
12. 

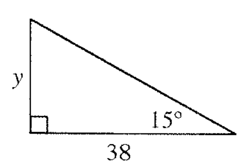
Use trigonometric ratios to solve for the variable in each figure below.

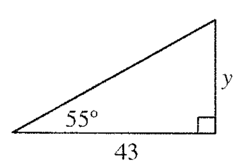
13. 

14. 

15. 

16. 

17. 

18. 

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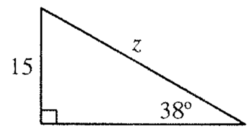

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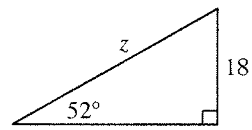

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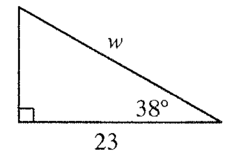

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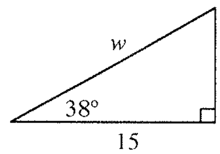

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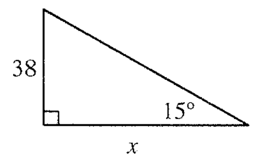

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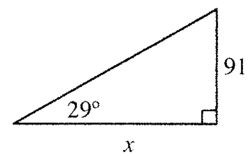

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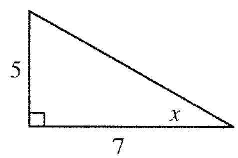

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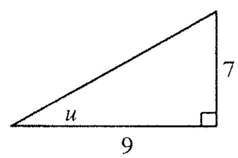

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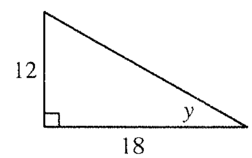

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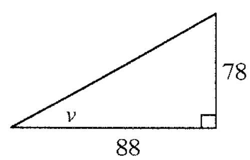


Draw a diagram and use trigonometric ratios to solve each of the following problems.

29. Juanito is flying a kite at the park and realizes that all 500 feet of string are out. Margie measures the angle of the string with the ground with her clinometer and finds it to be 42.5°. How high is Juanito's kite above the ground?

30. Nell's kite has a 350 foot string. When it is completely out, Ian measures the angle to be 47.5°. How far would Ian need to walk to be directly under the kite?

31. Mayfield High School's flagpole is 15 feet high. Using a clinometer, Tamara measured an angle of 11.3° to the top of the pole. Tamara is 62 inches tall. How far from the flagpole is Tamara standing?

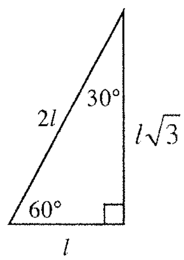
32. Tamara took another sighting of the top of the flagpole from a different position. This time the angle is 58.4°. If everything else is the same, how far from the flagpole is Tamara standing?

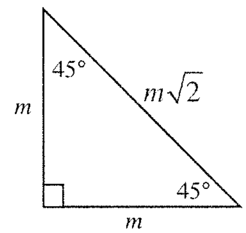
33. Standing 140 feet from the base of a building, Alejandro uses his clinometer to site the top of the building. The reading on his clinometer is 42°. If his eyes are 6 feet above the ground, how tall is the building?

34. An 18 foot ladder rests against a wall. The base of the ladder is 8 feet from the wall. What angle does the ladder make with the ground?

SPECIAL RIGHT TRIANGLES 5.2.1 – 5.2.2

There are two special right triangles that occur often in mathematics: the 30°-60°-90° triangle and the 45°-45°-90° triangle. By AA ~, all 30°-60°-90° triangles are similar to each other, and all 45°-45°-90° triangles are similar to each other. Consequently, for each type of triangle, the sides are proportional. The sides of these triangles follow these patterns.





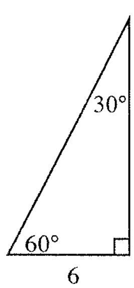
Another short cut in recognizing side lengths of right triangles are Pythagorean Triples. The lengths 3, 4, and 5 are sides of a right triangle (Note: You can verify this with the Pythagorean Theorem) and the sides of all triangles similar to the 3-4-5 triangle will have sides that form Pythagorean Triples (6-8-10, 9-12-15, etc.). Another common Pythagorean Triple is 5-12-13.

See the Math Notes boxes in Lessons 5.2.1 and 5.3.1.

Example 1

The triangles below are either a 30°-60°-90° triangle or a 45°-45°-90° triangle. Decide which it is and find the lengths of the other two sides based on the pattern for that type of triangle.

a.



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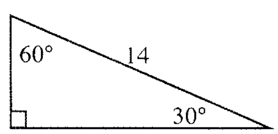

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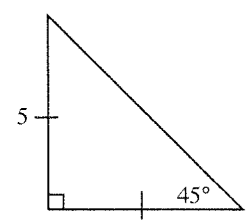

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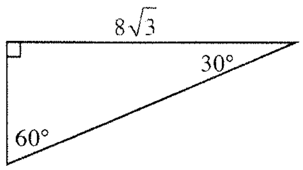

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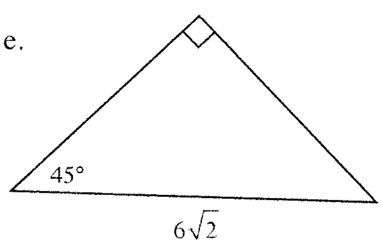

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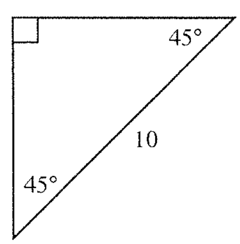
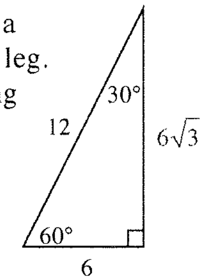
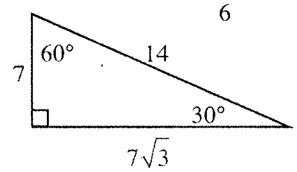


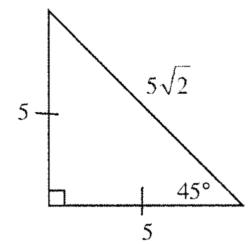
In part (a), this is a 30°-60°-90° triangle, so its sides will fit the pattern for such a triangle. The pattern tells us that the hypotenuse is twice the length of the short leg. Since the short leg has a length of 6, the hypotenuse has a length of 12. The long leg is the length of the short leg times $\sqrt{3}$, so the long leg has a length of $6\sqrt{3}$.



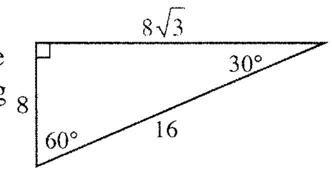
In part (b), we have a 30°-60°-90° triangle again, but this time we know the length of the hypotenuse. Following the pattern, this means the length of the short leg is half the hypotenuse: 7. As before, we multiply the length of the short leg by $\sqrt{3}$ to get the length of the long leg: $7\sqrt{3}$.



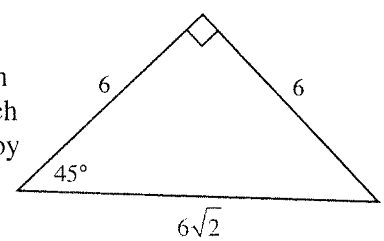
The triangle in part (c) is a 45°-45°-90° triangle. The missing angle is also 45°; you can verify this by remembering the sum of the angles of a triangle is 180°. The legs of a 45°-45°-90° triangle are equal in length (it is isosceles) so the length of the missing leg is also 5. To find the length of the hypotenuse, we multiply the leg's length by $\sqrt{2}$. Therefore the hypotenuse has length $5\sqrt{2}$.



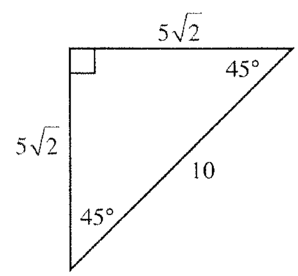
We have another 30°-60°-90° triangle in part (d). This time we are given the length of the long leg. To find the short leg, we *divide* the length of the long leg by $\sqrt{3}$. Therefore, the length of the short leg is 8. To find the length of the hypotenuse, we double the length of the short leg, so the hypotenuse is 16.



The triangle in part (e) is a 45°-45°-90° triangle, and we are given the length of the hypotenuse. To find the length of the legs (which are equal in length), we will divide the length of the hypotenuse by $\sqrt{2}$. Therefore, each leg has length 6.



If you understand what was done in each of the previous parts, part (f) is no different from the rest. This is a 45°-45°-90° triangle, and we are given the length of the hypotenuse. However, we are used to seeing the hypotenuse of a 45°-45°-90° triangle with a $\sqrt{2}$ attached to it. In the last part when we were given the length of the hypotenuse, we divided by $\sqrt{2}$ to find the length of the legs, and this time we do the same thing.



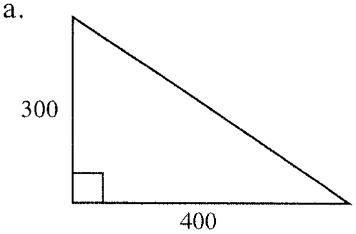
$$\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Note: Multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$ is called rationalizing the denominator. It is a technique to remove the radical from the denominator.

Example 2

Use what you know about Pythagorean Triples and similar triangles to fill in the missing lengths of sides below.

a.



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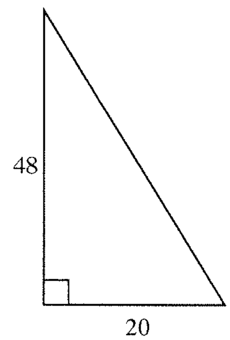
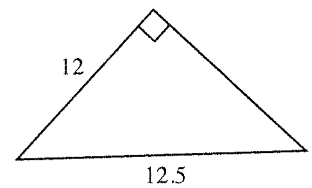


c.

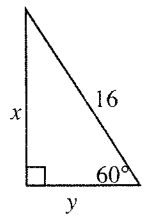


There are a few common Pythagorean Triples that students should recognize; 3–4–5, 5–12–13, 8–15–17, and 7–24–25 are the most common. If you forget about a particular triple or do not recognize one, you can always find the unknown side by using the Pythagorean Theorem if two of the sides are given. In part (a), this is a multiple of a 3–4–5 triangle. Therefore the length of the hypotenuse is 500. In part (b), we might notice that each leg has a length that is a multiply of four. Knowing this, we can rewrite them as 48 = 4(12), and 20 = (4)(5). This is a multiple of a 5–12–13 triangle, the multiplier being 4. Therefore, the length of the hypotenuse is 4(13) = 42. In part (c), do not let the decimal bother you. In fact, since we are working with Pythagorean Triples and their multiples, double both sides to create a similar triangle. This eliminates the decimal. That makes the leg 24 and the hypotenuse 25. Now we recognize the triple as 7–24–25. Since the multiple is 0.5, the length of the other leg is 3.5.

Problems

Identify the special triangle relationships. Then solve for x, y, or both.

1.



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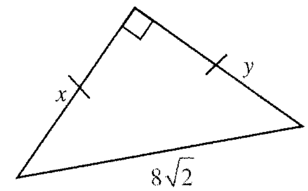

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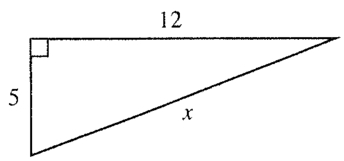

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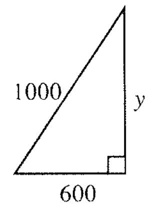

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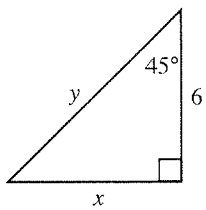

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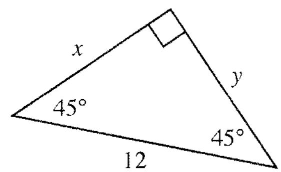

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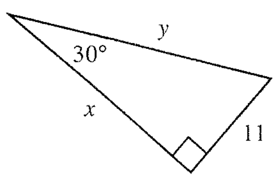

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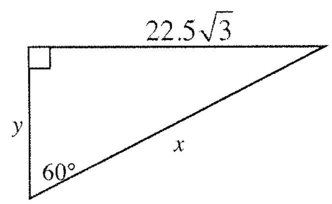

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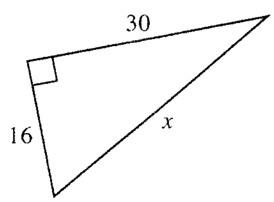

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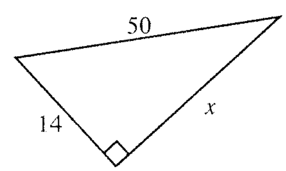
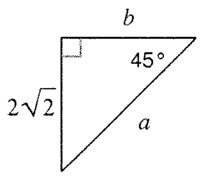


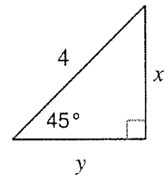
Kuta Software - Infinite Geometry Name________________________

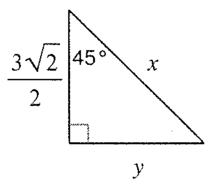
Special Right Triangles

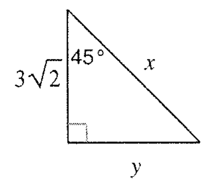
Date________________ Period____

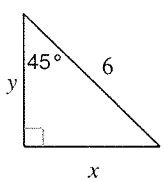
Find the missing side lengths. Leave your answers as radicals in simplest form.

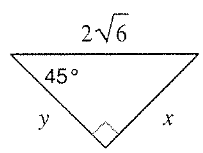
1) 

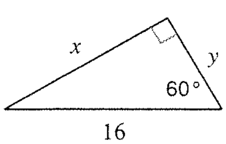
2) 

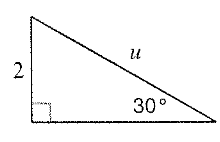
3) 

4) 

5) 

6) 

7) 

8) 

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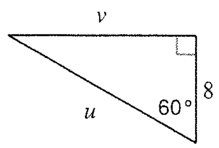

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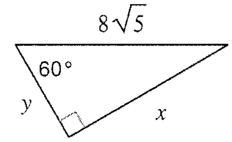

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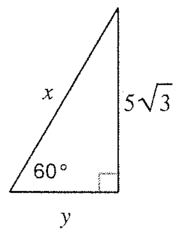

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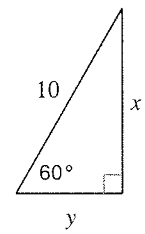

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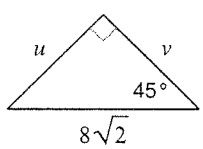

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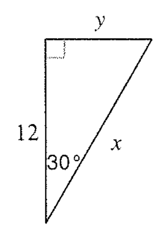

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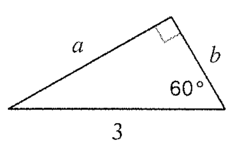

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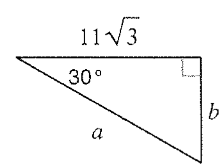

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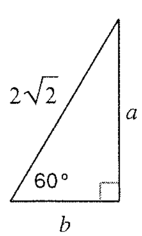

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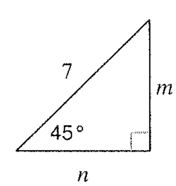


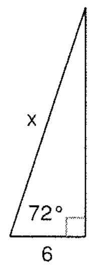
Kuta Software - Infinite Geometry Name________________________________

Solving Right Triangles

Date________________ Period____

Find the missing side. Round to the nearest tenth.

1)



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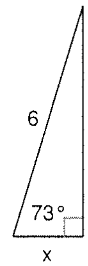

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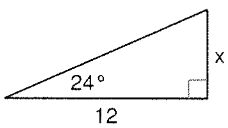

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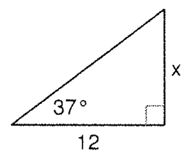

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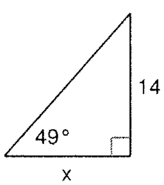

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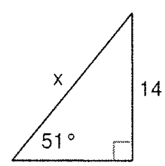

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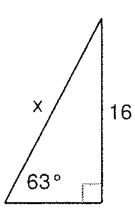

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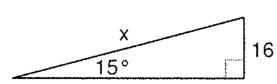

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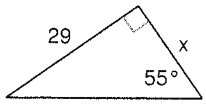

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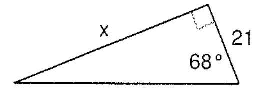

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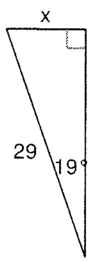

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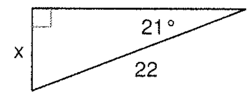

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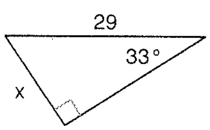

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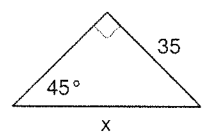

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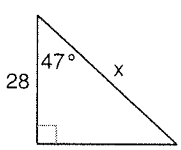

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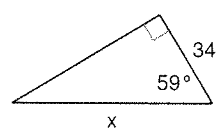


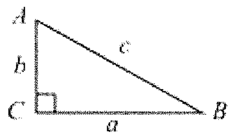
Critical thinking question:

17) Write a new problem that is similar to the others on this worksheet. Solve the question you wrote.

NAME______________________________ DATE________________________ CLASS____

ACT MATH SKILLS PREP – TRIGONOMETRY

1. For the right triangle ΔABC shown below, what is $\cos B$?



A. $\frac{b}{c}$

B. $\frac{a}{c}$

C. $\frac{b}{a}$

D. $\frac{c}{b}$

E. $\frac{c}{a}$

3. If $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, then $\cot\theta = ?$

A. $-\frac{4}{5}$

B. $-\frac{5}{4}$

C. $\frac{3}{4}$

D. $-\frac{3}{4}$

E. $-\frac{4}{3}$

2. If $\sin A = \frac{1}{\sqrt{5}}$, and $\tan A = \frac{1}{2}$, then $\cos A = ?$

A. $\frac{2}{\sqrt{5}}$

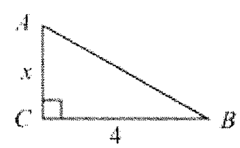
B. $\frac{\sqrt{5}}{2}$

C. 2

D. $\sqrt{5}$

E. $2\sqrt{5}$

4. In right triangle ΔABC as shown below, which of the following is an expression for $\sin A$?



A. $\frac{\sqrt{x^2+16}}{4}$

B. $\frac{\sqrt{x^2+16}}{x}$

C. $\sqrt{x^2+16}$

D. $\frac{4}{\sqrt{x^2+16}}$

E. $\frac{x}{\sqrt{x^2+16}}$

NAME____ DATE________________________________ CLASS_____

ACT MATH SKILLS PREP – TRIGONOMETRY

5. If $0° < x < 90°$ and $\cos x = \frac{8}{17}$, then $\sin x = ?$

A. $\frac{17}{8}$

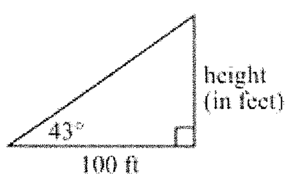
B. $\frac{8}{15}$

C. $\frac{15}{8}$

D. $\frac{17}{15}$

E. $\frac{15}{17}$

6. The distance from a point on the ground to a flag pole is 100 feet. The angle of elevation from the point on the ground to the top of the pole is $43°$. What is the height of the pole to the nearest tenth of an inch?



A. 68.2

B. 73.1

C. 93.3

D. 107.2

E. 146.6

7. For all real numbers *x*, which of the following expressions is equivalent to $\cos^2 x + \sin^2 x$?

A. 0

B. 1

C. 6

D. 12

E. $\tan^2 x$

8. For values of *x* where $\sin x$, $\cos x$, and $\tan x$ are all defined, $\frac{(\sin x)(\cos x)}{(\tan x)} = ?$

A. $\frac{\cos^2 x}{\sin x}$

B. $\frac{\cos x}{\sin x}$

C. 1

D. $\sin^2 x$

E. $\cos^2 x$