Students Note:

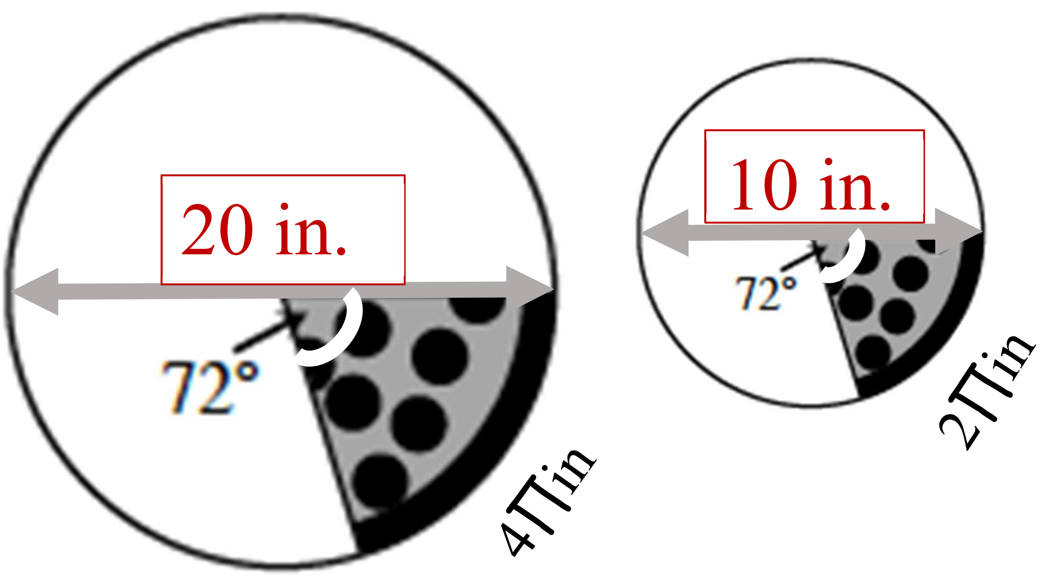
1) ***When you finish reading this lesson, you'll be able to complete the Fun Sheets titled:***
   a) Arcs, Central Angles & Inscribed Angles
   b) Going Around in Circles
   c) Circles & Central Angles
   d) Central Angles & Inscribed Angles
   e) Central Angles & Inscribed Angles – ACT Problems

2) ***If you need extra help, videos are available on Algebra Nation:***
   Section 10, Topic 1 pg 269, & Topic 2 pg 271

1) Arc Length vs. Arc Measure

You'll recall that, in our previous lesson, we learned how to find the Length of Arc (the "crust" of the pizza slice.) Well now, we're going to learn how to find the *Measure* of an Arc. (Huh?! What?!! I thought we did that already!)

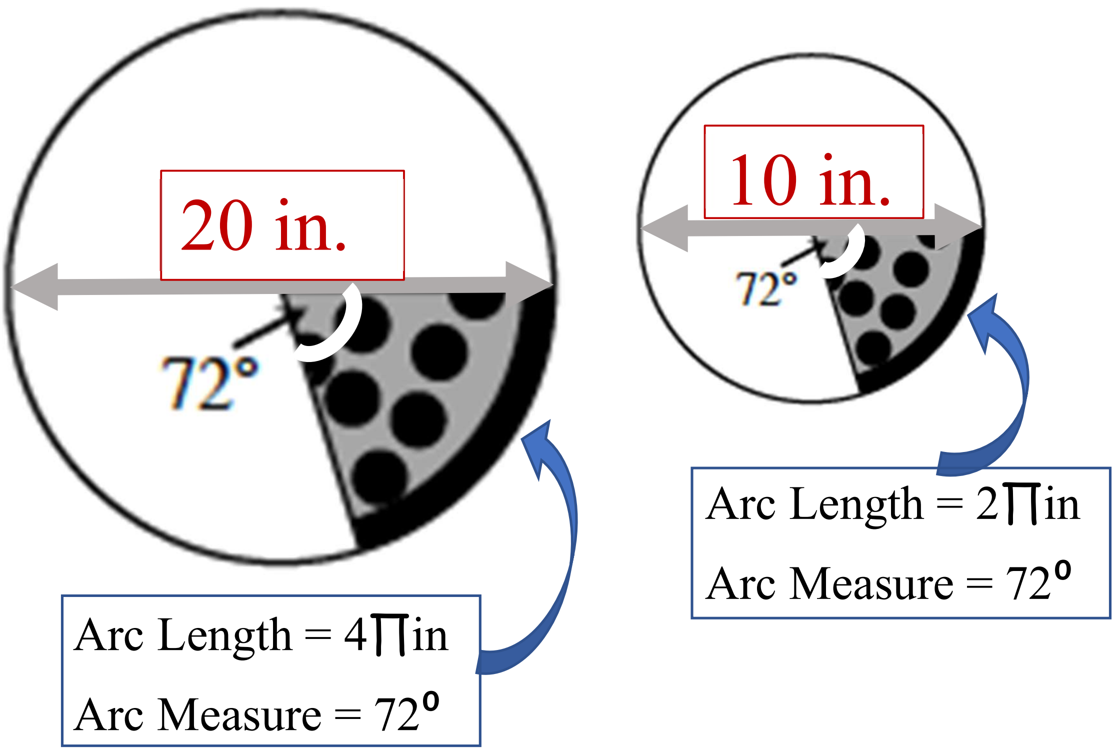
Well, there's actually a difference between what we call "Arc Length" and "Arc Measure." Take, for example, the two pizzas below:



One pizza is large (the kind Blade likes), while the other is small. Notice that, even though the Central Angle on each is the same (72º), the crust length on the slice shown on the left is 4∏in, while the one shown on the right is half of that -- 4∏in. – Which makes sense, right? Because the smaller pizza's diameter is half that of the larger pizza, it makes sense that the crust length of the smaller slice would be half the crust length of the larger.

However, even though the Arc Lengths are different, the Arc Measures are still the same – both Arc *Measures* are 72º -- the same measure as the Central Angle.

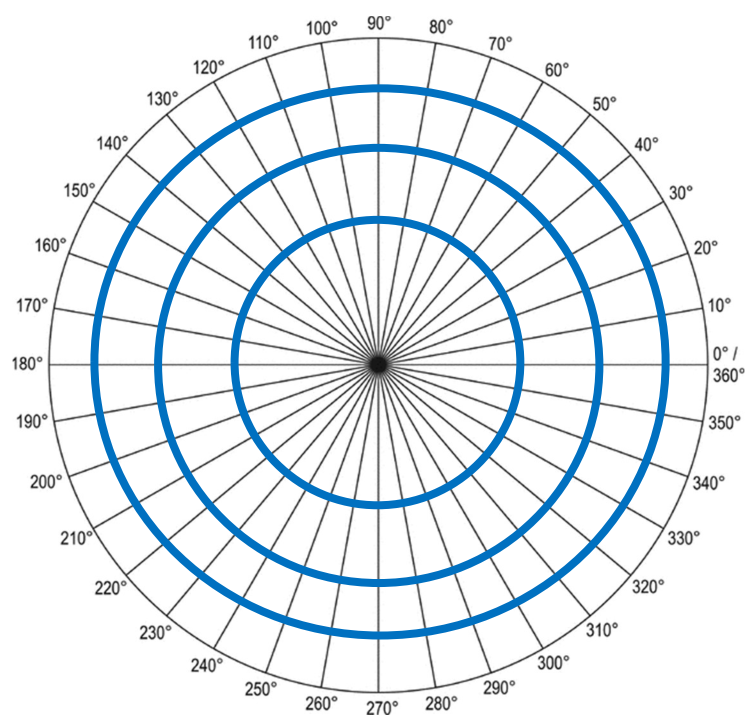
Arc *Length* is measured in *Linear Units* (e.g., inches, feet, centimeters), while Arc *Measure* is measured in *Degrees.*



Let's look at the diagram at right, for instance.

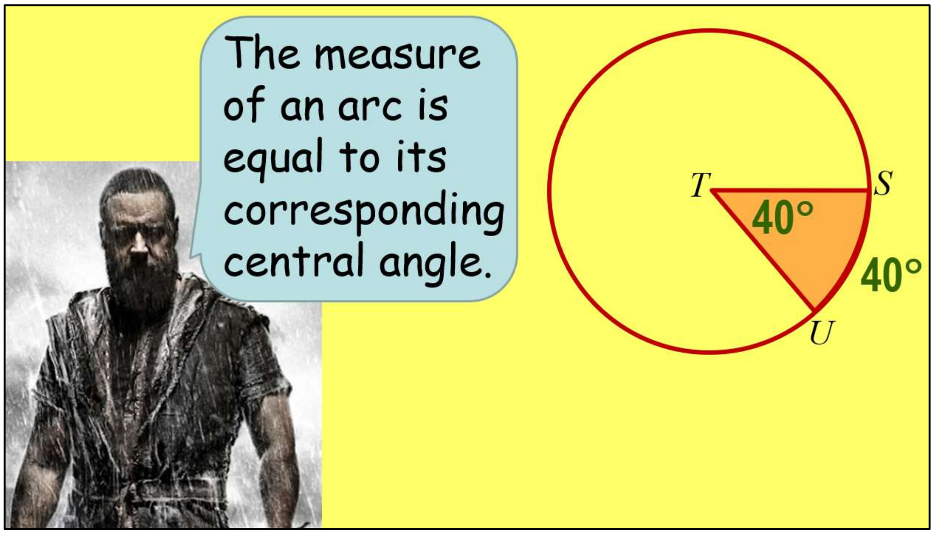
Notice that the number of degrees between each of the radii shown – the *arc measure* -- is 10°. So, the *Arc Measure* between each of the radii shown is 10°, regardless of which circle we're talking about – it could be the smallest circle shown, or the largest – the *measure* is going to be the same.

However, if we're talking about Arc *Length* – Take a look at the distance between the radii on the outer circle, and then look at the distance between the radii on the inner-most circle. You can see that the lengths of the arcs are different, the same way the lengths of the Circumferences are also different. (Even though the # of degrees in any Circle is 360°, regardless of its size.)

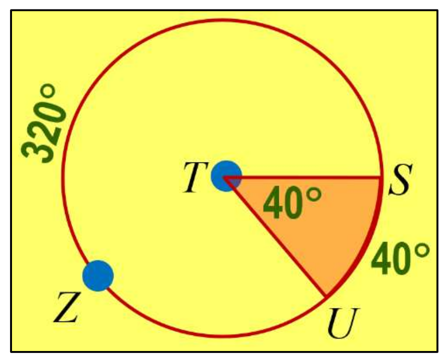


2) <u>Central Angles, Inscribed Angles, & Arcs</u>

So, we've talked a bit about Central Angles and Arcs. There are other types of Angles in a Circle, as well. Let's listen to what our guide, Noah (who we know is an expert in Ar[k]s), has to say about these things, and the relationships between them:



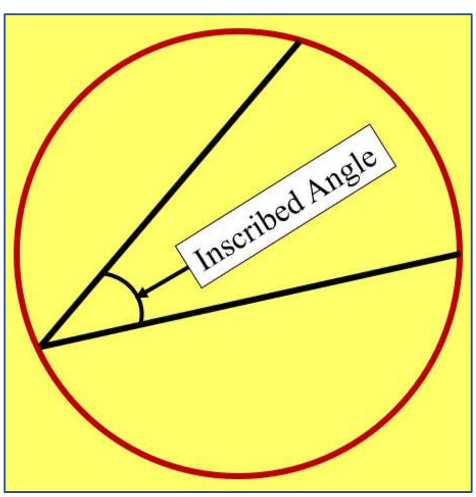
So, as mentioned above, there is a difference between Arc *Measure* & Arc *Length.* In Noah's example at left, he's referring to the *Measure* of the Arc – *not* its *Length* – and he's stating that the *Measure* of the Arc is equal to its corresponding Central Angle.



Now, we know that *Arc SU* measures 40°. But what of the other arc – the *Major Arc?* (Recall from a previous lesson that, if two arcs are created as shown, the smaller arc is called the *Minor Arc* – named with 2 letters, while the larger arc is called the *Major Arc* – named with 3 letters.)

Well, if we know that *Minor Arc SU* measures 40, and we know that the entire circle measures 360, then to find the measure of the *Major Arc SZU,* we just subtract 40 from 360 to get the measure of the *Major Arc:*

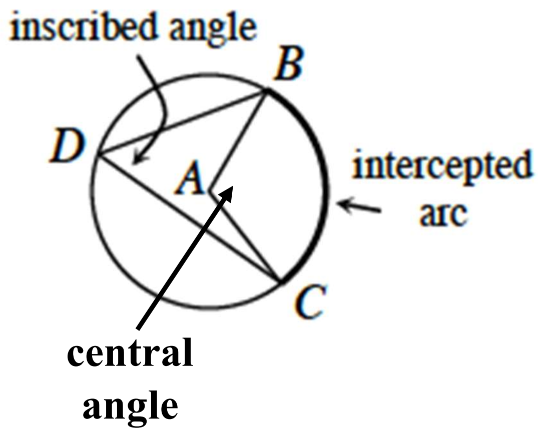
$$360 - 40 = 320$$



<u>Inscribed Angles</u>

Another type of angle we might find in a Circle is called an *Inscribed Angle.* (See diagram at left.) Notice that an Inscribed Angle has its vertex on the Circumference of the Circle.

There is a relationship between an Inscribed Angle and its Intercepted Arc (the arc between the rays of the angle – see diagram below), the same way there's a relationship between a Central Angle and its Intercepted Arc, as shown above. However, instead of the angle being equal to the *Intercepted arc,* the *Inscribed Angle* is 1/2 the measure of the *Intercepted Arc.*

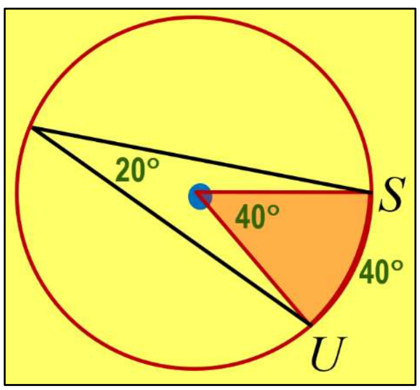


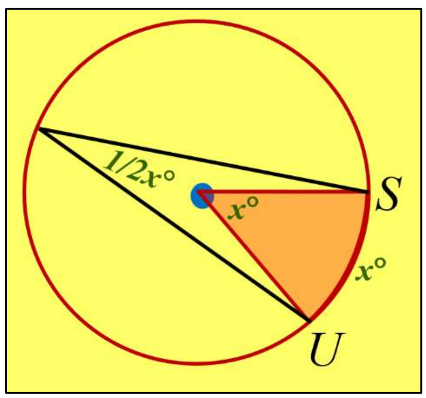
- An ***Inscribed Angle*** has its vertex on the Circumference of the Circle.

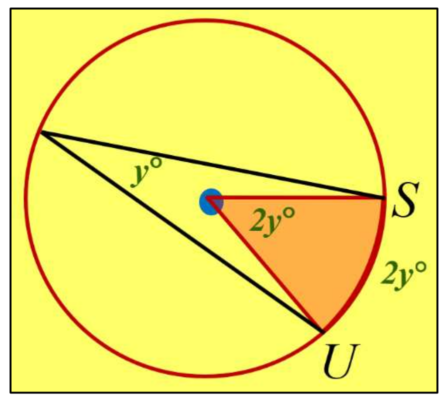
- A ***Central Angle*** has its vertex in the Center of the Circle.

- An ***Intercepted Arc*** is the arc between the rays that form the angle.

So, if an *Inscribed Angle* is ½ the measure of its *Intercepted Arc,* and if a *Central Angle* is equal to the measure of the same Intercepted Arc, then that means an *Intercepted Arc* is also 1/2 the measure of a *Central Angle* that shares the same *Intercepted Arc*. (As shown in the following diagrams.)

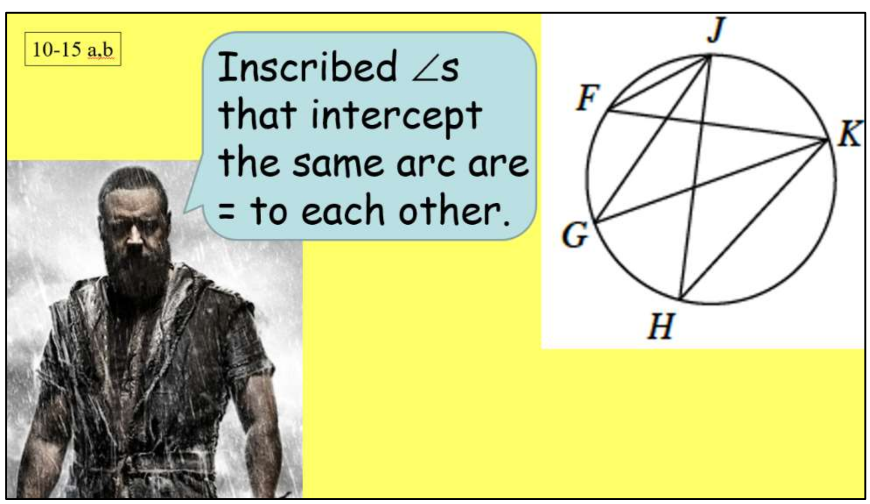


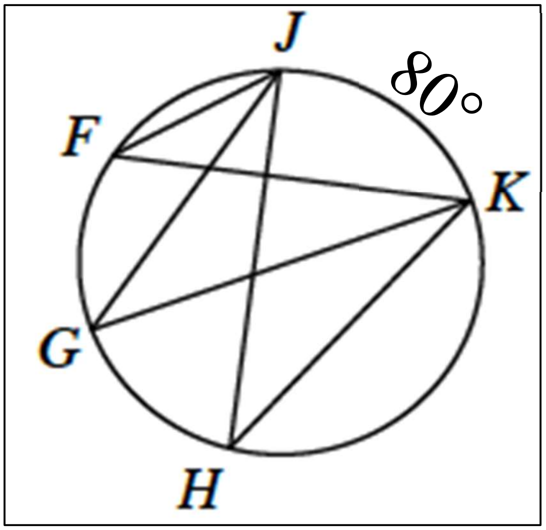




Another interesting thing about Inscribed Angles:

Notice that Angles *F, G* & *H* all intercept the same arc – *Arc JK.* When this happens, the Inscribed Angles are equal to each other.





So if, for instance, Intercepted Arc JK measured 80°, what would Angles F, G & H each measure?

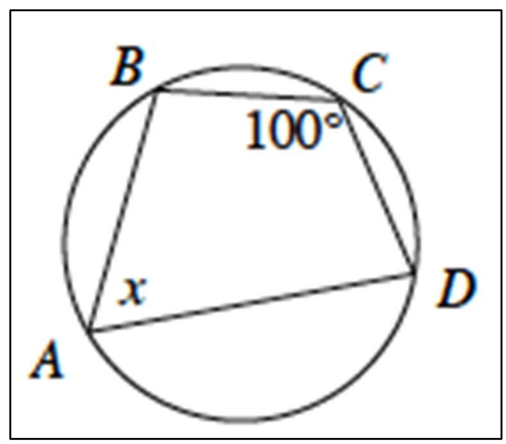
Well if we know that:

1) Inscribed Angles are 1/2 the measure of their Intercepted Arc; and

2) Inscribed Angles that intercept the same arc are equal to each other;

Then we know that Angles F, G & H are each going to measure 40°!

Quadrilaterals Inscribed in Circles



When we know about things like Inscribed Angles, we can figure out other interesting stuff. Take, for instance, the diagram at right. It shows a quadrilateral inscribed in a circle.

We want to find the measure of *"x"*. How would we do that?

Well, we know that angle C measures 100°. So, what else can we figure out based on that? Glad you asked.

Notice that Inscribed Angle C intercepts Arc BAD. So, what would that arc measure? Well, it's gotta measure 200°, because an Inscribed Angle is ½ the measure of its Intercepted Arc. (Conversely, an Intercepted Arc is 2x the measure of its corresponding Inscribed Angle.)

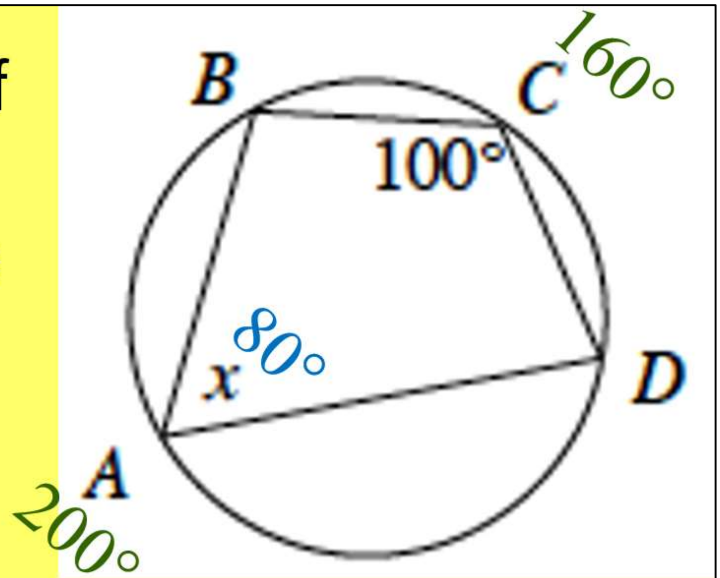
Now, what does Arc BCD measure? Well the entire Circle measures 360°, so we'll subtract 200 from 360 to find that Arc BCD measures 160°.

Now notice that Inscribed Angle BAD (which is the one we're solving for…), intercepts the arc that measures 160°. So, what does that Inscribed Angle have to measure? Well, it's ½ of 160 = 80°.

Now notice – what's the relationship between Angles BCD & BAD? That's right! – They're Supplementary!

This will happen whenever a quadrilateral is inscribed inside a circle. See the next page for our theorem…

The opposite ∠s of a quadrilateral inscribed in a circle are always supplementary.



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Students:

1) You should now be ready to work on Fun Sheets titled:

- ***a) Arcs, Central Angles & Inscribed Angles***
- ***b) Going Around in Circles***
- ***c) Circles & Central Angles***
- ***d) Central Angles & Inscribed Angles***
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A different kind of Ark… ⟶