DeSoto
COUNTY SCHOOLS

Semester
Algebra I

Week 3

LAWS OF EXPONENTS 3.1.1 and 3.1.2

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic **laws of exponents** are listed here.

(1) $x^a \cdot x^b = x^{a+b}$ Examples: $x^3 \cdot x^4 = x^7$; $2^7 \cdot 2^4 = 2^{11}$

(2) $\frac{x^a}{x^b} = x^{a-b}$ Examples: $\frac{x^{10}}{x^4} = x^6$; $\frac{2^4}{2^7} = 2^{-3}$

(3) $(x^a)^b = x^{ab}$ Examples: $(x^4)^3 = x^{12}$; $(2x^3)^5 = 2^5 \cdot x^{15} = 32x^{15}$

(4) $x^0 = 1$ Examples: $2^0 = 1$; $(-3)^0 = 1$; $\left(\frac{1}{4}\right)^0 = 1$

(5) $x^{-n} = \frac{1}{x^n}$ Examples: $x^{-3} = \frac{1}{x^3}$; $y^{-4} = \frac{1}{y^4}$; $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(6) $\frac{1}{x^{-n}} = x^n$ Examples: $\frac{1}{x^{-5}} = x^5$; $\frac{1}{x^{-2}} = x^2$; $\frac{1}{3^{-2}} = 3^2 = 9$

(7) $x^{m/n} = \sqrt[n]{x^m}$ Examples: $x^{2/3} = \sqrt[3]{x^2}$; $y^{1/2} = \sqrt{y}$

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 3.1.2. For additional examples and practice, see the Checkpoint 5A problems in the back of the textbook.

Example 1

Simplify: $(2xy^3)(5x^2y^4)$

Reorder: $2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$

Using law (1): $10x^3y^7$

Example 2

Simplify: $\frac{14x^2y^{12}}{7x^5y^7}$

Separate: $\left(\frac{14}{7}\right) \cdot \left(\frac{x^2}{x^5}\right) \cdot \left(\frac{y^{12}}{y^7}\right)$

Using laws (2) and (5): $2x^{-3}y^5 = \frac{2y^5}{x^3}$

Example 3

Simplify: $(3x^2y^4)^3$

Using law (3): $3^3 \cdot (x^2)^3 \cdot (y^4)^3$

Using law (3) again: $27x^6y^{12}$

Example 4

Simplify: $(2x^3)^{-2}$

Using law (5): $\dfrac{1}{(2x^3)^2}$

Using law (3): $\dfrac{1}{2^2 \cdot (x^3)^2}$

Using law (3) again: $\dfrac{1}{4x^6}$

Example 5

Simplify: $\dfrac{10x^7y^3}{15x^{-2}y^3}$

Separate: $\left(\dfrac{10}{15}\right) \cdot \left(\dfrac{x^7}{x^{-2}}\right) \cdot \left(\dfrac{y^3}{y^3}\right)$

Using law (2): $\dfrac{2}{3}x^9y^0$

Using law (4): $\dfrac{2}{3}x^9 \cdot 1 = \dfrac{2}{3}x^9 = \dfrac{2x^9}{3}$

Problems

Simplify each expression. Final answers should contain no parentheses or negative exponents.

1. $y^5 \cdot y^7$
2. $b^4 \cdot b^3 \cdot b^2$
3. $8^6 \cdot 8^{-2}$
4. $(y^5)^2$
5. $(3a)^4$
6. $\frac{m^8}{m^3}$
7. $\frac{12m^8}{6m^{-3}}$
8. $(x^3y^2)^3$
9. $\frac{(y^4)^2}{(y^3)^2}$
10. $\frac{15x^2y^5}{3x^4y^5}$
11. $(4c^4)(ac^3)(3a^5c)$
12. $(7x^3y^5)^2$
13. $(4xy^2)(2y)^3$
14. $\left(\frac{4}{x^2}\right)^3$
15. $\frac{(2a^7)(3a^2)}{6a^3}$
16. $\left(\frac{5m^3n}{m^5}\right)^3$
17. $(3a^2x^3)^2(2ax^4)^3$
18. $\left(\frac{x^3y}{y^4}\right)^4$
19. $\left(\frac{6x^8y^2}{12x^3y^7}\right)^2$
20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$
21. x^{-3}
22. $2x^{-3}$
23. $(2x)^{-3}$
24. $(2x^3)^0$
25. $5^{1/2}$
26. $\left(\frac{2x}{3}\right)^{-2}$

Using Exponents

Name ______________________

Date ______________ Period ______

Simplify.

1. $\frac{a^3}{a^2}$ a

2. $\frac{b^4}{b}$

3. $\frac{ab^7}{ab^2}$

4. $\frac{c^3d^4}{c^2d}$

5. $\frac{12x^4y^7}{2x^2y^2}$

6. $\frac{-10m^5n^3}{5mn}$

7. $\frac{2a^{10}b^5}{8a^5b^2}$

8. $\frac{-13x^{15}y^{11}}{39x^7y^{10}}$

9. $\frac{-8x^2y^4}{16x^2y^7}$

10. $\frac{6x^3y^7}{24x^5y^9}$

11. $\frac{-15x^5y}{45xy^3}$

12. $\frac{30a^4b^2}{60a^2b^7}$

13. $\frac{(2a^3)^2}{4a^2}$

14. $\frac{-(3x^2)^3}{9x^4}$

15. $\frac{27a^4}{(3a^2)^4}$

16. $\frac{25x^4y^7}{(10x^2y)^2}$

17. $\frac{26(a^3b^6)^2}{39(a^3b^4)^3}$

18. $\frac{15(r^2t^5)^3}{45(r^3t^2)^4}$

19. $\frac{a^4}{a^r}$

20. $\frac{x^5}{x^t}$

21. $\frac{8x^m}{(2x)^2}$

22. $\frac{(2m^2)^4}{8m^r}$

23. $\frac{(a^2b^3)^r}{(a^4b)^r}$

24. $\frac{10^5}{10^2}$

25. $\frac{2^9}{2^3}$

26. $\frac{3^7}{3^2}$

27. $\frac{7^{10}}{7^5}$

28. $\frac{11^6}{11^3}$

29. $\frac{(2a^3b^4)^3}{(2ab^2)^5}$

30. $\frac{(3x^5y^3)^5}{(6x^{10}y^7)^2}$

31. $\frac{(2x^ry^t)^2}{3x^5y^7}$

32. $\frac{(2a^mb^t)^3}{5a^6b^2}$

33. $\frac{7a^sb^v}{11a^rb^t}$

34. $\frac{(2x^4y^5)^r}{(2x^3y^2)^r}$

35. $\frac{(3a^7b^2)^m}{(3a^2b^5)^m}$

36. $\frac{(10c^3d^7)^p}{(10c^6d^3)^p}$

Two ways to find the area of a rectangle are: as a product of the (height) · (base) or as the sum of the areas of individual pieces of the rectangle. For a given rectangle these two areas must be the same, so **area as a product = area as a sum**. Algebra tiles, and later, generic rectangles, provide area models to help multiply expressions in a visual, concrete manner.

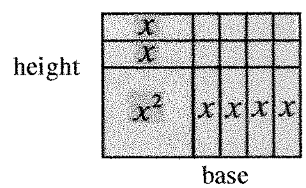
For additional information, see the Math Notes boxes in Lessons 3.2.2, 3.2.3, and 3.3.3. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

Example 1: Using Algebra Tiles

The algebra tile pieces $x^2 + 6x + 8$ are arranged into a rectangle as shown at right. The area of the rectangle can be written as the **product** of its base and height or as the **sum** of its parts.

$$\underbrace{(x+4)}_{\text{base}}\underbrace{(x+2)}_{\text{height}} = \underbrace{x^2+6x+8}_{\text{area}}$$

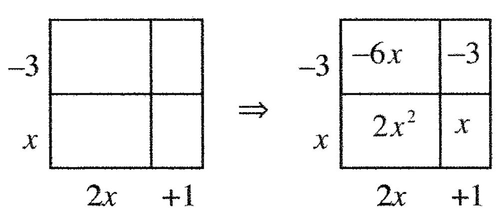
area as a **product** area as a **sum**



Example 2: Using Generic Rectangles

A generic rectangle allows us to organize the problem in the same way as the first example without needing to draw the individual tiles. It does not have to be drawn accurately or to scale.

Multiply $\underbrace{(2x+1)}_{\text{base}}\underbrace{(x-3)}_{\text{height}}$.

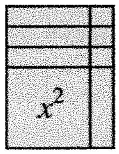


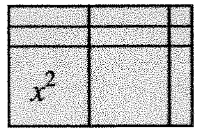
$\Rightarrow (2x + 1)(x - 3) = 2x^2 - 5x - 3$

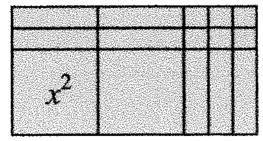
area as a product area as a sum

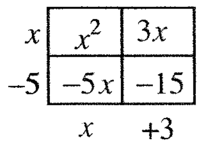
Problems

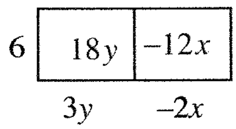
Write a statement showing **area as a product** equals **area as a sum**.

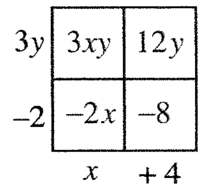
1. 

2. 

3. 

4. 

5. 

6. 

Multiply.

7. $(3x + 2)(2x + 7)$
8. $(2x - 1)(3x + 1)$
9. $(2x)(x - 1)$
10. $(2y - 1)(4y + 7)$
11. $(y - 4)(y + 4)$
12. $(y)(x - 1)$
13. $(3x - 1)(x + 2)$
14. $(2y - 5)(y + 4)$
15. $(3y)(x - y)$
16. $(3x - 5)(3x + 5)$
17. $(4x + 1)^2$
18. $(x + y)(x + 2)$
19. $(2y - 3)^2$
20. $(x - 1)(x + y + 1)$
21. $(x + 2)(x + y - 2)$

SOLVING EQUATIONS WITH MULTIPLICATION OR ABSOLUTE VALUE 3.3.1

To **solve an equation with multiplication**, first use the Distributive Property or a generic rectangle to rewrite the equation without parentheses, then solve in the usual way. For additional information, see the Math Notes box in Lesson 3.3.1. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

To **solve an equation with absolute value**, first break the problem into two cases since the quantity inside the absolute value can be positive or negative. Then solve each part in the usual way.

Example 1

Solve $6(x + 2) = 3(5x + 1)$

Use the Distributive Property.	$6x + 12 = 15x + 3$
Subtract $6x$.	$12 = 9x + 3$
Subtract 3.	$9 = 9x$
Divide by 9.	$1 = x$

Example 2

Solve $x(2x - 4) = (2x + 1)(x + 5)$

Rewrite the equation using the Distributive Property on the left side of the equal sign and a generic rectangle on the right side.

$2x^2 - 4x =$

$+5$	$10x$	5
x	$2x^2$	x
	$2x$	$+1$

$$2x^2 - 4x = 2x^2 + 11x + 5$$

Subtract $2x^2$ from both sides.	$-4x = 11x + 5$
Subtract $11x$ from both sides.	$-15x = 5$
Divide by -15.	$x = \frac{5}{-15} = -\frac{1}{3}$

Example 3

Solve $|2x-3|=7$

Separate into two cases.	$2x-3=7$	or	$2x-3=-7$
Add 3.	$2x=10$	or	$2x=-4$
Divide by 2.	$x=5$	or	$x=-2$

Problems

Solve each equation.

1. $3(c+4)=5c+14$
2. $x-4=5(x+2)$
3. $7(x+7)=49-x$
4. $8(x-2)=2(2-x)$
5. $5x-4(x-3)=8$
6. $4y-2(6-y)=6$
7. $2x+2(2x-4)=244$
8. $x(2x-4)=(2x+1)(x-2)$
9. $(x-1)(x+7)=(x+1)(x-3)$
10. $(x+3)(x+4)=(x+1)(x+2)$
11. $2x-5(x+4)=-2(x+3)$
12. $(x+2)(x+3)=x^2+5x+6$
13. $(x-3)(x+5)=x^2-7x-15$
14. $(x+2)(x-2)=(x+3)(x-3)$
15. $\frac{1}{2}x(x+2)=\left(\frac{1}{2}x+2\right)(x-3)$
16. $|3x+2|=11$
17. $|5-x|=9$
18. $|3-2x|=7$
19. $|2x+3|=-7$
20. $|4x+1|=10$

REWRITING MULTI-VARIABLE EQUATIONS 3.3.2

Rewriting equations with more than one variable uses the same "legal" moves process as solving an equation with one variable in Lessons 3.2.1, A.1.8, and A.1.9. The end result is often not a number, but rather an algebraic expression containing numbers and variables.

For "legal" moves, see the Math Notes box in Lesson 3.2.1. For additional examples and more practice, see the Checkpoint 6A materials at the back of the textbook.

Example 1

Solve for y	$3x-2y=6$
Subtract $3x$	$-2y=-3x+6$
Divide by –2	$y=\frac{-3x+6}{-2}$
Simplify	$y=\frac{3}{2}x-3$

Example 2

Solve for y	$7+2(x+y)=11$
Subtract 7	$2(x+y)=4$
Distribute the 2	$2x+2y=4$
Subtract $2x$	$2y=-2x+4$
Divide by 2	$y=\frac{-2x+4}{2}$
Simplify	$y=-x+2$

Example 3

Solve for x	$y=3x-4$
Add 4	$y+4=3x$
Divide by 3	$\frac{y+4}{3}=x$

Example 4

Solve for t	$I=prt$
Divide by pr	$\frac{I}{pr}=t$

Problems

Solve each equation for the specified variable.

1. Solve for y: $5x+3y=15$
2. Solve for x: $5x+3y=15$
3. Solve for w: $2l+2w=P$
4. Solve for m: $4n=3m-1$
5. Solve for a: $2a+b=c$
6. Solve for a: $b-2a=c$
7. Solve for p: $6-2(q-3p)=4p$
8. Solve for x: $y=\frac{1}{4}x+1$
9. Solve for r: $4(r-3s)=r-5s$

