Basic integration rules, integrating polynomials

Consider taking the derivative of a single term of a polynomial:

$f(x) = 5x^3$

We did this by *multiplying by the power* (3) and then **subtracting 1 from the power**.

Working **backwards** on this process is known as **integration** or taking the **antiderivative**.

Specifically, for the problem above, we would begin with the answer $(15x^2)$,

- raise the power by 1 and
- then divide by the new power.

This is almost right. What if the original problem had been to take the derivative of $f(x) = 5x^3 + 7$ or perhaps $f(x) = 5x^3 + 10$? In all cases the derivative would have still been $f'(x) = 15x^2$.

To account for the constant that could clearly be anything, here are the modified rules for integrating a term of a polynomial:

- raise the power by 1,
- divide by the new power, and
- add an arbitrary constant (call it C)

The **symbolism for integrating** a function, f(x), is $\int \mathbf{f(x)dx}$.

Integration rule summary:

- $\int 0dx = C$ (Recall that the derivative of a constant is 0.)
- $\int dx = x + C$ (Think of the function as $f(x) = x^0$.)
- $\int k\, f(x)dx = k \int f(x)dx$ (*k* is a constant)
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ where $n \neq -1$
  (This even works when *n* is negative or fractional.)

Example 1: $\int 6x^2 dx = ?$

Example 2: $\int -11x^6 dx = ?$

Example3: $\int (4x^5 - 12x + 1)\, dx = ?$

Example 4: $4\int x^{-2}dx = ?$

Example 5: $\int \left(7\sqrt{x} + \frac{1}{x^2}\right) dx = ?$

The integrals introduced in this lesson are known as **indefinite integrals** and are denoted with just a plain integral sign ... $\int f(x)dx$.

Definite integrals that we will learn about later, have an upper and lower limit and are denoted like this... $\int_2^{11} f(x)dx$.

1. $f(x) = x$

2. $f(x) = 5$

3. $g(x) = 7x^5 - 36x^2 + x$

4. $y = 1/x^2$

5. $f(x) = \sqrt[3]{x}$

6. $f(x) = (x + 5)^2$

7. $g(x) = (\sqrt{x} + 8)/x^3$

8. $f(x) = (5/6)x^4 - 8x^3 + x^2 - 3x + 9$

Let's try some problems that involve using an initial value to determine C.

Find f(x) where $f''(x) = x^3 - x^2$, $f'(2) = -1$, and $f(1) = 3$.

Find $f(x)$ where $f''(x) = x^2 - x$, $f(1) = -3$, and $f(0) = 4$.

An object is moving along a straight line with a velocity given by $v(t) = 3t^2 - 6t + 1$. Find both the acceleration and position when the object is at $s = 5$ when $t = 2$.

An object is moving along a straight line with a velocity given by $v(t) = 3t - 6t^2$. Find both the acceleration and position when the object is at position 0 at $t = 2$.