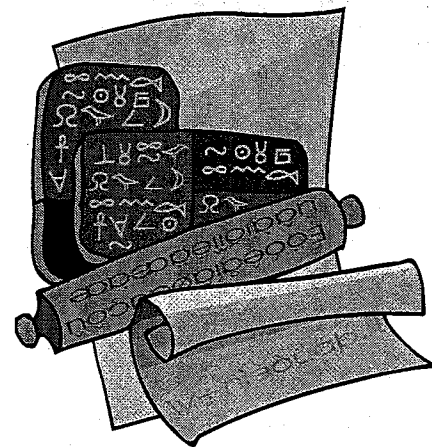


The Language of Algebra

Section 4.1

Key Words in Word Problems



When given the symbols of mathematics, most of us can easily recognize and interpret them. But when you put those same symbols and operations into “words,” that’s not so easy. There are some key words or phrases that provide important *clues* as to how the math is done. Let’s review some of these key words and phrases below.

Key Words

Words or Phrases	Example	Translation
Addition		
<i>sum</i>	the <i>sum</i> of sixteen and twenty-three is thirty-nine	$16 + 23 = 39$
<i>increased by</i>	five <i>increased by</i> three is eight	$5 + 3 = 8$
<i>plus</i>	four <i>plus</i> ten is fourteen	$4 + 10 = 14$
<i>more than</i>	three <i>more than</i> six is nine	$3 + 6 = 9$
<i>added to</i>	ten <i>added to</i> five is fifteen	$10 + 5 = 15$
<i>total</i>	the <i>total</i> of twenty and thirteen is thirty-three	$20 + 13 = 33$
Subtraction		
<i>difference of</i>	the <i>difference of</i> sixteen and twelve is four	$16 - 12 = 4$
<i>decreased by</i>	ten <i>decreased by</i> three is seven	$10 - 3 = 7$
<i>minus</i>	fifteen <i>minus</i> ten is five	$15 - 10 = 5$
<i>less than</i>	two <i>less than</i> seventeen is fifteen	$17 - 2 = 15$
<i>subtracted from</i>	twenty-two <i>subtracted from</i> forty-four is twenty-two	$44 - 22 = 22$
<i>reduced by</i>	eighteen <i>reduced by</i> four is fourteen	$18 - 4 = 14$
Multiplication		
<i>product of</i>	the <i>product of</i> ten and thirty is three hundred	$10 \times 30 = 300$
<i>multiplied by</i>	sixteen <i>multiplied by</i> three is forty-eight	$16 \cdot 3 = 48$
<i>times</i>	fifteen <i>times</i> four is sixty	$15(4) = 60$
<i>of, percent of</i>	ten <i>percent of</i> one hundred is ten	$0.10 (100) = 10$
Division		
<i>quotient of</i>	the <i>quotient of</i> ten and two is five	$10 \div 2 = 5$
<i>divided by</i>	sixteen <i>divided by</i> four is four	$16 \div 4 = 4$
<i>per, out of</i>	3 <i>out of</i> 4 (3 <i>per</i> 4) of 28 is 21.	$\frac{3}{4}(28) = 21$
<i>percent (divide by 100)</i>	15 <i>percent</i> is equal to 15 divided by 100.	$15\% = 15 \div 100$
<i>the ratio of</i>	<i>the ratio of</i> 1 to 4 is 0.25	$\frac{1}{4} = 0.25$

Section 4.1, continued

Key Words in Word Problems

Did you notice that the translations given are all equations written with an equal sign? Which word in each of the examples gives the *clue* that the translation should be an equation? If you said, “*is*,” you are right. The word *is* in a word problem will usually mean *equals*. Of course, you may also see the word *equals*, which of course should be interpreted as an equal sign. You may remember from English class that words like *is* and *equals* are verbs. In math word problems, verbs are often your clue to put in an equals sign.

Algebra Word Problems

When it comes to algebra word problems, you will always have one or more unknown values. Those *unknowns* are assigned variables. Setting up algebra expressions or equations from words is the first and most important step towards solving the problem. If the words are not translated into the right expressions or equations, you will not be able to solve the problem correctly. Algebra word problems also have key words that you must recognize. Here are the steps to follow when setting up algebra word problems.

Steps to Setting Up Word Problems

1. Interpret what is being asked.
2. Identify what is given.
3. Write down your unknowns and assign variables to them.
4. Write down any assumed knowledge (not always needed).
5. Write down the mathematical relationships given in terms of the variables by using clues and key words from the problem.
6. Use the mathematical relationships to answer the question.

Example 1: Josh has three more dollars than his brother Billy. If B represents the number of dollars that Billy has, what expression represents the number of dollars that Josh has?

Step 1: What is being asked?

To find an expression for the number of dollars Josh has

Step 2: What is given?

That Josh has three more dollars than Billy, B

Step 3: What are the unknowns?

Let B = Number of dollars that Billy has
Let J = Number of dollars that Josh has

The problem tells you to use the variable, B , to represent the number of dollars that Billy has, so you should immediately recognize that B is a variable for an unknown value.

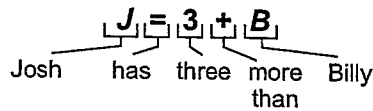
However, the number of dollars that Josh has is also unknown. You are to write an expression for how much Josh has. You can assign this unknown value a variable of J .

Step 4: What does this problem assume that you know?

This problem assumes that you know what an *expression* is. You will handle this assumption in step 6.

Section 4.1, continued
Key Words in Word Problems

Step 5: What mathematical relationships are given?



The relationship given in this example is fairly simple. The verb *has* in this example gives you the equals sign. The term *more than* tells you to add.

Step 6: Answer the question.

$$3 + B$$

The question asked for the expression that represents the amount of money Josh has. Remember, an expression is just one side of an equation and does not include the equal sign.

Example 2: Lisa is four less than three times as old as her sister, Kate. Write an equation that shows Lisa's age L in terms of Kate's age, K .

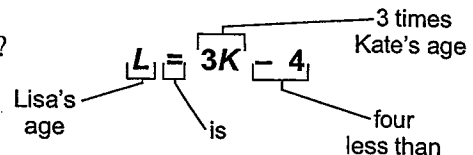
Step 1: What is being asked? Equation for Lisa's age in terms of Kate's age

Step 2: What is given? Lisa is four less than three times as old as Kate

Step 3: What are the unknowns? Let L = Lisa's age
Let K = Kate's age

Step 4: What does this problem assume that we know? No assumed knowledge

Step 5: What mathematical relationships are given?



Notice that the *four less than* comes after the $3K$. *Less than* means *subtracted from*, so it must come after the term from which you are subtracting. *Writing "4 - 3K" would be wrong!*

Step 6: Answer the question. $L = 3K - 4$

Since the question asks for an equation in terms of L and K , this relationship is the answer.

Example 3: Riley purchased 6 bags of potato chips for a party. Before tax was added, the total cost of the 6 bags was equal to the cost of 1 bag plus \$5.00. Write an equation that could be used to determine c , the cost of one bag of potato chips.

Step 1: What is being asked? An equation in terms of c , the cost of one bag of potato chips.

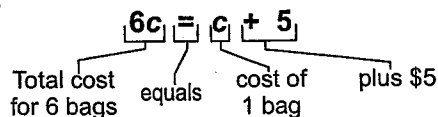
Section 4.1, continued
Key Words in Word Problems

Step 2: What is given? **total cost of 6 bags is equal to cost of 1 bag plus \$5.00**

Step 3: What are the unknowns? **Let c = the cost of one bag**

Step 4: What does this problem assume that you know? **The total cost of 6 bags is $6c$ (the cost of one bag times 6)**

Step 5: What mathematical relationships are given?



Step 6: Answer the question. **$6c = c + 5$**

This equation answers the question. You could use this equation to find c , the cost of one bag of potato chips.

Practice

Practice translating the following words into mathematical equations or expressions. Write the equation or expression in the blank.

1. Alexi has 4 more video games than Jason. Write an expression that shows the number of video games Alexi has as compared to the number of games Jason has, J .

2. Six less than a number, a , is four times another number, b . Write an equation that expresses the relationship between the numbers a and b .

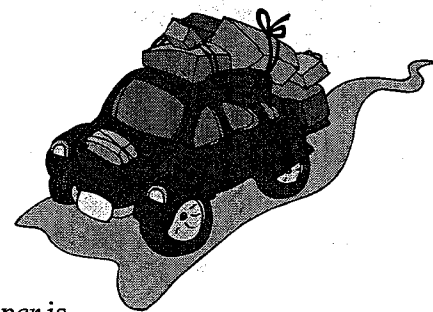
3. Oliver purchased 3 hotdogs at the baseball game. Before tax was added, the total cost of the 3 hotdogs was equal to the cost of 2 hotdogs plus \$5. Write an equation that could be used to determine c , the cost of 1 hotdog.

4. Maria saved 15 percent of her paycheck, p . Write an expression for the amount she saved in terms of p .

5. Mr. Hart's current water bill shows that he used 40 gallons less than 3 times the amount of water he did last month. If w is the amount he used last month, write an expression for the amount he used this month.

The Language of Algebra

Section 4.2 Rate Problems



Many types of word problems involve rates. Rates are given as “one unit per another unit.” For example, miles per gallon, dollars per hour, and dots per square inch are all rates. Rates are actually fractions. The first “unit” is the top number of the fraction. The “other unit” is the bottom number of the fraction. The *per* is the fraction bar between the two. If only one quantity is given, the second is assumed to be equal to one.

$$34 \text{ miles per gallon} = \frac{34 \text{ miles}}{1 \text{ gallon}}$$

$$\$7.00 \text{ per hour} = \frac{7 \text{ dollars}}{1 \text{ hour}}$$

Understanding Units

When a rate is given in a word problem, look to see what units are in the answer. Normally, the answer will need to be in one of the units from the rate. For example, if you are giving miles per gallon in a word problem, the solution will probably need to be in miles or gallons.

In algebra problems, units can be treated like variables. So in a word problem, how do you get from miles per gallon to miles? You multiply by gallons. Since units are like variables, they cancel like variables because they’re common terms. Remember, a number divided by itself is always one, and a variable divided by itself is also equal to one.

$$\frac{3}{3} = 1 \quad \frac{a}{a} = 1 \quad \frac{\text{miles}}{\text{miles}} = 1$$

Solving Rate Problems

If you will keep track of the units by treating them like variables, you will rarely make a mistake when solving word problems with units. Let’s look at some examples.

Example 1: Jason’s rental car gets 24 miles per gallon. The tank holds 18 gallons. How many miles can he travel on one tank of gas?

In this problem, you are given a rate in miles per gallon. The solution will be in miles. You know that a rate is a fraction, but where do the numbers go? By understanding units, you do not have to guess or even think too hard about whether to multiply or divide. The units will tell you. Just make sure the units for the answer end up in the numerator (top of the fraction).

- Step 1:** Write the rate in fraction form. This gives you a fraction with miles in the numerator and gallons in the denominator.
- Step 2:** The second quantity given is 18 gallons. The rate has gallons in the denominator. That means you will need gallons in a numerator to cancel out the gallons term. Multiply the rate by 18 gallons over one.
- Step 3:** The gallon units cancel out, and you are left with miles. And that’s great since the question asks for the answer in miles.

$$\frac{24 \text{ miles}}{1 \text{ gallon}} \times \frac{18 \text{ gallons}}{1} = \frac{24 \text{ miles} \times 18 \text{ gallons}}{1 \text{ gallon}} = 432 \text{ miles}$$

Section 4.2, continued

Rate Problems

Remember, the unit in the answer must be on top. Let's look at this another way.

Example 2: Jason's rental car gets 24 miles per gallon. If he has traveled 168 miles, how many gallons has he used?

This time, you are given miles per gallon and miles, and the answer is in gallons. Again, the units will tell you what to do.

Step 1: Here, you need gallons in the numerator because the answer is in gallons. To get gallons in the numerator, reverse the order of the rate. The rate, 24 miles per gallon, can be restated as 1 gallon per 24 miles.

Step 2: The second quantity is 168 miles. The rate now has miles in the denominator, so you need a miles unit in the numerator to cancel out the miles. Therefore, multiply the rate by the miles or 168 miles over one.

Step 3: The miles units cancel out, and you are left with gallons. Do the division, and get the answer.

$$\frac{1 \text{ gallon}}{24 \text{ miles}} \times \frac{168 \text{ miles}}{1} = \frac{1 \text{ gallon} \times 168 \text{ miles}}{24 \text{ miles}} = 7 \text{ gallons}$$

Example 3: Susan makes \$7.00 per hour. How much will she make in x hours?

The same rules apply even when one of the quantities is a variable. In this case, the answer is $7x$.

$$\frac{7 \text{ dollars}}{1 \text{ hour}} \times \frac{x \text{ hours}}{1} = 7x \text{ dollars.}$$

Example 4: Susan makes \$7.00 per hour. How many hours will it take for her to earn x dollars?

Always rearrange the rate so that the units for the answer are in the numerator.

$$\frac{1 \text{ hour}}{7 \text{ dollars}} \times \frac{x \text{ dollars}}{1} = \frac{x \text{ hours}}{7} \text{ or } \frac{x}{7} \text{ hours}$$

Practice

Solve the following rate problems by cancelling units. Show your work. Write your answer in the blank.

1. Mrs. Hendricks used 25 gallons of gas on a business trip. Her car averages 25 miles per gallon. How many miles did she travel?

2. Ruben goes to the hardware to buy nails. The nails he needs cost \$0.35 per pound. Excluding sales tax, if he buys seven pounds of nails, how much in dollars will he pay for his purchase?

Section 4.2, continued
Rate Problems

3. A certain laser printer has a resolution of 600 dpi, which means it will print 600 dots per inch. If the printer printed an image 3,000 dots wide, how many inches wide is the image?
- _____
4. The scale on the map is 150 miles per inch. If Atlanta and Los Angeles are 15 inches apart on the map, how many miles is it to Los Angeles from Atlanta?
- _____
5. The posted speed limit is 65 miles per hour. After $5\frac{1}{2}$ hours of driving, how many miles have you gone?
- _____
6. The scale on the blueprint shows one inch per four feet. If the bench is three feet long, how many inches is that on the blueprint?
- _____
7. Sound travels approximately one mile in 4.8 seconds. From the time you saw the lightning flash until you heard the thunder was 24 seconds. How far in miles were you away from the lightning?
- _____
8. Ribbon for a sewing project cost \$0.08 per inch. How many inches of ribbon can you get for one dollar?
- _____
9. The weather report said there had been 6 inches of rain in the last 24 hours. At that rate, how much rain would there be in the next four hours?
- _____

The Language of Algebra

Section 4.3 Using Rates in Equations



If the word problem involves a rate, you can use what you know about rates to check your units. Remember, you can only add and subtract quantities that have the same type of units.

Example 1: A company charges a flat fee of \$5.00 to ship a package plus \$0.50 per each pound the package weighs. Write an equation that represents the total cost, T , for shipping a package weighing x pounds.

Step 1: What is being asked?

To find an equation for the total cost for shipping a package

Step 2: What is given?

Flat fee is \$5 plus \$0.50 per pound of package weight

Step 3: What are the unknowns?

Let T = Total cost the company charges in dollars

Let x = package weight in pounds

The problem tells you to use T to represent the total cost and x for the package weight.

Step 4: What mathematical relationships are given?

$$T = 5 + 0.5x$$

Diagram illustrating the components of the equation $T = 5 + 0.5x$:

- T : total charges
- 5 : \$5.00 flat fee
- $+$: plus
- 0.5 : \$0.50 per pound
- x : pounds

Unit analysis for the term $0.5x$:

$$\frac{\$0.5}{\text{pound}} \times \frac{x \text{ pounds}}{1} = \$0.5x$$

To get the total charges, the problem says the company adds \$5.00 and \$0.50 per pound for each pound. The tricky part of this problem is realizing you must multiply the rate per pound by the number of pounds to get dollars.

Step 5: Answer the question.

$$T = 5 + 0.5x$$

Since the question asks for an equation for the total cost, T , in terms of x , this is the equation. Notice that both the terms of 5 and $0.5x$ are in dollars.

Section 4.3, continued

Using Rates in Equations

Practice 1

Answer each of the problems below.

1. Temika's telephone plan charges \$25 per month for basic service plus \$0.08 per minute for long distance calls. If T equals the total monthly charge and L equals long distance minutes, what is the equation that represents the total charge for one month?

2. Some teachers are paid a base salary plus a bonus for every year of teaching experience. The base salary is \$19,500, and the "experience" bonus is \$1,095 per year of experience. What is the equation that represents the yearly salary, Y , of a teacher with N years experience?

3. The cost of buying apples in bulk is \$0.50 per pound plus a shipping and handling charge of \$10. Write an expression to represent the cost of x pounds of apples.

4. A laser printer takes 2 minutes to warm up, but after warming-up, it will print 16 pages per minute. Write an expression to represent the number of minutes it would take to print a document of p pages. (Caution: pay close attention to the units.)

5. A copier repair worker charged a customer \$760 for on-site copier repair and maintenance. The charge was based on a fixed service charge of \$190 plus \$95 for each hour spent servicing the copier. Write an equation that could be used to calculate, h , the number of hours spent on servicing the copier.

6. When catering a party, a caterer charges \$12 per person plus a \$100 set-up fee. If the total charges were \$520, write an equation that could be used to find p , the number of people at the party.

7. During the summer, Howard makes \$8.50 per hour working for his uncle's construction business. Write an equation that could be used to find the number of hours, x , that Howard must work to earn \$510.

Section 4.3, continued

Using Rates in Equations

You may not automatically recognize a problem as a rate problem, but when you see a *per*, it indicates a rate. Now consider a problem with two different “rates.” These types of problems are similar, but when problems have two (or more) different rates, the total is the sum of the parts. The general equation for two rates is given below.

$$\text{Total} = (\text{rate 1}) \times (\text{number of items at rate 1}) + (\text{rate 2}) \times (\text{number of items at rate 2})$$

Example 2: Jacob purchased 15 packages of cookies for a birthday party. Some of the cookies cost \$2.50 per package and others cost \$1.75 per package. Write an equation that represents the total cost of all 15 packages of cookies if he purchased x packages at \$2.50 each.

Step 1: What is being asked?

To find an equation for the total cost of cookies

Step 2: What is given?

A total of 15 packages

Some cost \$2.50 per package, so rate 1 is 2.5

The others cost \$1.75 per package, so rate 2 is 1.75

Step 3: What are the unknowns?

Let C = Total cost the cookies

Let x = number of packages that cost \$2.50

You don't know how many packages of cookies were purchased at each price, but the problem tells you that x number of packages were purchased at \$2.50 each.

Step 4: What knowledge is assumed?

Let $15 - x$ = number of packages that cost \$1.75

The number of packages purchased at \$1.75 must be the total, 15, minus the unknown number, x . This problem assumes that you can figure that out!

Step 5: What mathematical relationships are given?

$$\text{Total cost} = (\text{rate 1}) \times (\text{number of items at rate 1}) + (\text{rate 2}) \times (\text{number of items at rate 2})$$

$$C = 2.5x + 1.75(15 - x)$$

The mathematical relationship is the same as the one given above. The total is the sum of the parts.

Step 6: Answer the question.

$$C = 2.5x + 1.75(15 - x)$$

The question asks for an equation that represents the total cost.

Section 4.3, continued

Using Rates in Equations

Sometimes multiple rate problems are more easily solved by including all the units. Make sure one of the units in the rate cancels so that the units on both sides of the equation are the same. Remember that you can add only units that are alike.

Example 3: The Moore family drove M miles to Tampa, Florida, in 7 hours. During the first part of the trip, the speed limit was 60 miles per hour. During the second part of the trip, the speed limit was 70 miles per hour. If they drove exactly the allowed speed limit, write an equation that could be used to calculate x , the number of hours driven at 60 miles per hour.

This problem may look a lot harder than example 2, but it is actually worked exactly the same way. Do you recognize this as a rate problem with two different rates? Write down everything you know and then substitute the values into the rate equation. Include all units and see how they cancel out.

Step 1: Write down everything you are given and assign variables to the unknowns.

Time = 7 hours
 Rate 1 = 60 miles per hour
 Rate 2 = 70 miles per hour

Let M = total miles
 Let x = number of hours driven at 60 miles per hour
 Let $7 - x$ = number of hours driven at 70 miles per hour

Step 2: Substitute the values into the rate equation.

$$\begin{array}{l} \text{Total} = (\text{rate 1}) \times (\text{number of items at rate 1}) + (\text{rate 2}) \times (\text{number of items at rate 2}) \\ \rightarrow \quad \quad \quad \rightarrow \quad \quad \quad \rightarrow \quad \quad \quad \rightarrow \quad \quad \quad \rightarrow \\ M = \frac{60 \text{ miles}}{\text{hour}} \cdot \frac{x \text{ hours}}{1} + \frac{70 \text{ miles}}{\text{hour}} \cdot \frac{(7 - x) \text{ hours}}{1} \end{array}$$

Can you see how a rate like “miles per hour” is treated the same as a rate like “dollars per item”? The “items” are hours. If you have set up the equation correctly, the unit in the denominator of the rates cancels with the unit for the number of items. In this case, the “hours” cancel. With the units canceled, the total in miles is equal to miles plus miles.

Step 3: Write the equation without the units.

$$M = 60x + 70(7 - x)$$

Example 4: Jeff’s car averages 22 miles per gallon while driving in the city but averages 30 miles per gallon while driving on the interstate. Jeff drives a total of 142 miles, some in the city and some on the interstate. Write an equation that shows how many gallons of gas, G , he uses if he drove x miles in the city.

Do you remember what you do to get the units of a rate to cancel if both the rate unit and the item unit have the same numerator? You flip the rate.

$$G = \frac{1 \text{ gallon}}{22 \text{ miles}} \cdot \frac{x \text{ miles}}{1} + \frac{1 \text{ gallon}}{30 \text{ miles}} \cdot \frac{(142 - x) \text{ miles}}{1}$$

Since G is in gallons, you need gallons in the numerator. Flip the rate so that miles is in the denominator and cancels the “miles” units.

$$G = \frac{x}{22} + \frac{142 - x}{30}$$

Section 4.3, continued

Using Rates in Equations

Practice 2

Answer each of the problems below.

1. The band sells candy as a fund raiser. The chocolate bars are \$2.00 each, and peanut rolls are \$1.50 each. In the first week, one band member sells a total of 25 pieces of candy. Write an equation that gives the total amount of money, T , that she raised that week if she sold x chocolate bars.

2. Jerry loves to exercise by jogging and riding his bike a total of 10 hours per week. He can jog 6 miles per hour and bike 9 miles per hour. Write an equation to show how many miles, M , Jerry exercises per week if he bikes x hours.

3. During the summer, Sean makes \$7.50 per hour cutting grass for his neighbors and \$5.25 per hour flipping hamburgers at a local restaurant. He usually works a total of 20 hours per week. Write an equation that shows the total amount that Sean earns per week, T , if he cuts grass for x hours and spends the rest of his 20 hours flipping burgers?

4. Jonah participates in a 6 mile "fun run" race for charity. During the race, he runs part of the way and walks the rest. He runs 5 miles per hour and walks 3 miles per hour. It takes him 1.5 hours to complete the race. If he walks x hours, write an equation that could be used to calculate how many miles he walks.

5. Naomi purchased 30 postage stamps, some for letters and some for postcards. The stamps for letters cost \$0.41 each. The stamps for postcards cost \$0.26 each. If Naomi purchased x postcard stamps, write an equation for the total cost of the stamps, c .

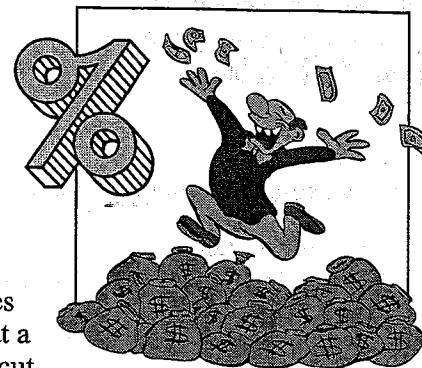
6. A color laser printer prints 6 pages per minute in color but prints 10 pages per minute in black and white. The printer prints for 30 minutes. Write an equation that would calculate the total number of pages, P , printed during that time if x minutes were needed for color pages.

7. A color laser printer prints 6 pages per minute in color but prints 10 pages per minute in black and white. A 32 page document contains x colored pages and the rest of the pages are in black and white. Write an equation that would calculate the time it would take the printer to complete the document. (*Be very careful with your units.*)

The Language of Algebra

Section 4.4

Setting up More Equations from Word Problems



Percentage Problems

Many types of word problems have percentages. Problems with commissions, sales tax, interest rates, tips, discounts, etc. usually mention percentages. Remember that a percentage must be converted to a decimal number by dividing by 100. (The shortcut for dividing by 100 is to move the decimal point *two places* to the left.)

$$10\% = \frac{10}{100} = 0.10$$

$$25\% = 0.25$$

$$5\% = 0.05$$

$$2.5\% = 0.025$$

$$110\% = 1.10$$

Caution: When converting a single digit percentage like 5% or 2.5% to a decimal, remember to add a zero in front of the percent so you can move two decimal places left. Don't worry about where the decimal starts; always move two places left.

Example 1: Roger wants to buy a computer priced at \$1,250. If sales tax is 7.5%, how much will he pay for the computer once the sales tax is added?

Sales tax is calculated by multiplying the sales tax percentage by the purchase price. Total cost is sales tax added to the purchase price.

$$\text{Total} = \underbrace{1,250}_{\text{purchase price}} + \underbrace{0.075 \times 1,250}_{\text{sales tax}}$$

The sales tax on this purchase is \$93.75. The total price with tax is \$1,343.75.

$$\text{Total} = 1,250 + 93.75 = 1,343.75$$

Example 2: Glen, the men's department manager at a department store, earns a weekly salary of \$550 plus a 5% commission of t , his total weekly sales. If his last week's total earnings was \$725, write an equation that could be used to determine his weekly sales?

- Step 1:** What is being asked? To find an equation for the total weekly sales.
- Step 2:** What is given? Weekly earnings are \$550 salary plus 5% commission of total sales
Weekly earnings last week totaled \$725
- Step 3:** What are the unknowns? Let t = total weekly sales
Let W = weekly earnings = \$725
- Step 4:** What does this problem assume that we know? 5% commission of t is calculated by $0.05t$.
- Step 5:** What mathematical relationships are given?
- $$725 = 550 + 0.05t$$
- Last week's total earnings
weekly salary
plus
5% commission of t , total weekly sales
- Step 6:** Answer the question. $725 = 550 + 0.05t$

The equation is the answer to the question. This equation could be used to calculate t , total weekly sales.

Section 4.4, continued

Setting up More Equations from Word Problems

Practice 1

Answer each of the following percentage problems.

1. Jim sees an item on an Internet auction site that is up for bids. The shipping instructions say to add 9% to the final bid for shipping. Write an equation for Jim to use to calculate total cost, T , with a final bid of b .

2. Your family and some friends go to the local steakhouse every Friday night. For parties of 6 or more, there is a mandatory gratuity (tip) of 15%. There are seven of you, and the total bill with the gratuity is \$78. Write an equation that could be used to calculate the amount of the meals, m , before the gratuity was added.

3. Your cell phone bill has to be paid by the 25th of each month or else there is a 1.5% late fee added to the bill. What equation would represent the total bill, T , if the late fee is added to the current charge, C .

4. If you are self-employed, you must pay our own taxes to the Internal Revenue Service. You have to “estimate” how much you will owe in taxes and send it in every three months. To make sure you have paid enough in estimated taxes, what you send must be equal to 110% of what you owed last year. Write a formula that will let you check to make sure you’ve paid enough. Let total paid, T , equal 110% of last year’s taxes, L .

Profit Problems

Profit is calculated by subtracting the cost of making or buying an item from the item’s sale price. If you make it, what you pay for the materials is called material cost (MC). If you buy it already made, it’s just cost (C).

Example 1: Ron makes fishing lures. The material cost is \$1.25 per lure. He sells each lure for \$3.75. How much profit does Ron make per lure?

Remember that profit is equal to sale price minus material cost. Sale price is \$3.75 and material cost is \$1.25. Therefore, once we do the math, profit is \$2.50 per lure.

$$\text{Profit} = \text{Sale Price} - \text{Material Cost}$$

$$\text{Profit} = \$3.75 - \$1.25$$

$$\text{Profit} = \$2.50$$

Example 2: Kara buys and resells books at a profit. If she pays \$2.95 for books that she sells for \$5.50, how much profit does Kara make if she sells 12 books?

This problem is calculated the same way, except this time, you must subtract what Kara paid, C , and then multiply the profit for one item by the number of items sold. In this case, we multiply by 12 to get a total profit of \$30.60

$$\text{Total Profit} = (\text{SP} - C) \times \text{Number of Items}$$

$$\text{Total Profit} = (\$5.50 - \$2.95) \times 12$$

$$\text{Total Profit} = \$2.55 \times 12 = \$30.60$$

Section 4.4, continued
Setting up More Equations
from Word Problems

Example 3: A copy shop charges customers \$0.10 per page to make copies. The material cost per copy is only \$0.01. The shop has to pay an additional \$150 per month for the service contract on the copier. Write an equation which could be used to determine x , the number of copies the shop must sell each month just to pay for the service contract.

Step 1: What is being asked? To find an equation that would determine the number of copies the shop must sell to pay for the \$150 service contract.

Step 2: What is given? Sale price per copy = \$0.10, Material cost per copy = \$0.01, Service contract = \$150 per month

Step 3: What are the unknowns? Let x = number of copies sold per month

Step 4: What does this problem assume that we know? Profit = Sale Price – Material Cost
 The shop uses profit to pay for service contract.

Step 5: What mathematical relationships are given?

$$150 = (0.10 - 0.01)x$$

Profit needed to pay service contract
Sale Price
Material Cost
Copies Sold per Month

Step 6: Answer the question. $150 = (0.10 - 0.01)x$ or $150 = 0.09x$

The equation is the answer to the question. This equation could be used to calculate x , the number of copies sold per month.

Practice 2

Answer each of the following profit problems. Show your work and write the final equation in the blank.

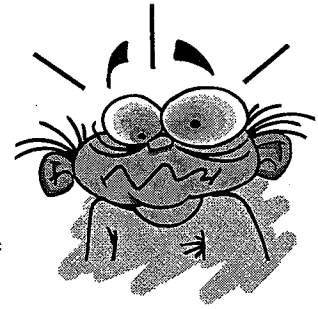
1. Carlos buys apples from the corner market and sells them in his neighborhood. He pays \$0.05 each and sells them for \$0.32 each. How much profit does he make if he sells 42 apples?

2. Dianne is making cookies for the bake sale. In all, she spent \$31.07 for the ingredients. If she uses all of the ingredients to make 25 dozen cookies, and she sells each cookie for \$0.25 each, how much profit does she make?

3. Casey and three friends buy watermelons from a farmer for \$1.12 each. They sell them at a school fund raiser for \$5 each. If their goal is to raise \$200 in profits for the fund raiser, write an equation that could be used to find the number of watermelons, w , they need to sell to meet their goal.

The Language of Algebra

Section 4.5 Reverse Word Problems



There may be times when you will be asked to recognize a word problem that would represent a given equation. In this case, analyze each possible answer to see which fits the given equation.

Example: Which of the following situations could be represented by the equation $y = 4x - 5$?

- A A car travels 4 miles in 5 minutes. At this rate, how many miles can the car travel in x minutes?
- B The Z-tron company offers its employees a 5% bonus four times a year based on their quarterly salary, x . What equation would show the yearly bonus total?
- C A teacher scores a test giving students four points for each question answered correctly and taking away 5 points for each question answered incorrectly. What is a student's score who answered x questions correctly?
- D A computer game gives a player four points for every target captured, but it automatically subtracts 5 points to begin the game. What is the total amount of points a player will score for x targets captured?

Can you eliminate two of the four answers immediately based on the fact that this problem involves subtraction? Answer choices A and B do not indicate any type of subtraction, so they cannot be the answers.



- A A car travels 4 miles in 5 minutes. At this rate, how many miles can the car travel in x minutes?
This is a rate problem. Remember, rate problems involve multiplication or division. Nothing else in this problem indicates subtraction.



- B The Z-tron company offers its employees a 5% bonus four times a year based on their quarterly salary, x . What equation would show the yearly bonus total?
This is a multiplication problem with no subtraction.



- C A teacher scores a test giving students four points for each question answered correctly and taking away 5 points for each question answered incorrectly. What is a student's score who answered x questions correctly?

the number of questions answered correctly = x

the number of questions answered incorrectly is also an unknown so it can be = z

a student's total score = y

$$y = 4x - 5z$$

This is not the correct answer because the unknown value of incorrect questions is not accounted for in the equation.



- D A computer game gives a player four points for every target captured, but it automatically subtracts 5 points to begin the game. What is the total amount of points a player will score for x targets captured?

targets captured = x

total points = y

$$y = 4x - 5$$

This is the correct answer.

Section 4.5, continued
Reverse Word Problems

Practice

Answer the following questions. Darken the circle that corresponds to the correct answer.

1. Which of the following situations could be represented by the equation $n = 6m - 4$?

- A CD's at the music store are \$6 each. Today there is a special sale that offers \$4 off any purchase. What is the total cost, n , for buying m cd's?
- B Boat rental at the lake is \$6 per hour for the first hour and \$4 per hour for each additional hour. What is the total rental cost, n , for renting a boat for m hours?
- C Shipping for an online purchase is \$6 per item plus a handling fee of \$4. What is the total shipping cost, n , for shipping m items?
- D The movie theater offers a \$4 discount for groups of 6 or more purchasing tickets in advance. What is the total charge, n , for a group of 6 or more if the ticket price is m dollars?

(A) (B) (C) (D)

3. Which of the following situations could be represented by the equation $r = 15s + 1$?

- A Teresa ordered 15 copies of a book for her class. The company offers a one dollar discount on every order. How much did she pay for the books if each book cost s dollars?
- B Larry is a martial arts instructor. For a class, he charges \$15 per student plus a one dollar per student registration fee. How much does he get paid for a class of s students?
- C Matt pays a flat rate of \$1 for a connection fee to make calls from his hotel room. If he makes 15 calls, what is the total, r , for phone calls on his bill.
- D A bus driver makes 15 trips a day to an off-site parking lot to pick up passengers. For each trip, he pays a toll of s dollars. The bus company reimburses him a total of r dollars for the money he spends on tolls plus a one dollar paperwork fee.

(A) (B) (C) (D)

2. Which of the following situations could be represented by the equation $Z = 6x^2$?

- A The area of a rectangle has a length of 6 and a width of x . What is the area, Z ?
- B The surface area of a cube has a side that measures x inches. What is the surface area, Z ?
- C The cube has sides equal to 6 inches each. What is the volume of the cube, Z ?
- D An equilateral triangle has sides equal to 6 inches each. What is the area of the triangle, Z ?

(A) (B) (C) (D)

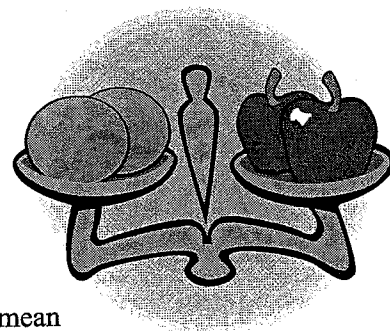
4. Which of the following situations could be represented by the equation $20 = (x + 1)(x)$?

- A The height of a cylinder is one inch greater than its diameter. If the volume of the cylinder is 20 in^3 , write an equation to find the height, x .
- B Four friends go out to eat together. The total bill is \$20. If 1 of them paid \$1 more than the others, how much did the others pay, x ?
- C The area of a rectangle is 20 ft^2 . Write an equation that could be used to find the length, x , if the width is 1 inch longer than the length.
- D The perimeter of a rectangle is 20 feet. Write an equation that could be used to find the length, x , if the width is 1 inch shorter than the length.

(A) (B) (C) (D)

The Language of Algebra

Section 4.6 Dimensional Analysis



Not all word problems involve variables. Some word problems require you to use previous knowledge and/or to convert from one type of unit to another.

Almost all word problems involve numbers with units. Without units, numbers don't mean much. If you asked someone to go get five, they would ask, "Five of what?" Do you want five apples, five pencils, five dollars, five feet, five what? The "what" is the unit. In algebra problems, units can be treated like variables. Units can be added, subtracted, multiplied, and divided. However, you still have to follow the rules of performing operations with variables. Adding $2x + 2y$ does not give you $4xy$ because they're not like terms. And adding 2 feet and 2 inches doesn't give you 4 feet-inches. They're not like units. To add feet and inches, you need to convert one unit to the other so the units will match.

Dimensional analysis is an easy way to convert units so that you don't have to worry about whether to multiply or divide. In dimensional analysis, you multiply by factors of one to convert from one unit to another. Remember, any value divided by itself will always be equal to one. That even applies if the units are not the same. For example, one foot is equal to twelve inches, right? So, if you multiply by a factor of one foot divided by twelve inches, you are multiplying by one. The units will cancel, just like they did with the rates. Let's see how it works. To make it easier to see how everything cancels, write all terms as fractions.

Example 1: From end zone to end zone, a football field measures 100 yards. How many inches are between the end zones?

$$\begin{array}{c}
 \text{given value} \quad \quad \quad \text{value of 1} \quad \quad \quad \text{value of 1} \\
 \hline
 \frac{100 \text{ yards}}{1} \times \frac{3 \text{ feet}}{1 \text{ yard}} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 100 \times 3 \times 12 \text{ inches} = 3600 \text{ inches}
 \end{array}$$

units in the numerator cancel units in the denominator

units of inches is the only units not cancelled (must be on top in the numerator)

Instead of writing out the multiplication signs, in dimensional analysis, you can use a shorthand notation by drawing vertical lines to separate the fractions multiplied together. To simplify things when you have multiple conversions, put the given units in the middle.

Example 2: The Thompson family traveled to Canada where mileage is measured in kilometers. They traveled 60 kilometers in 45 minutes. How many miles an hour were they traveling if 1 mile is approximately 1.6 kilometers.

$$\frac{1 \text{ mile}}{1.6 \text{ kilometers}} \mid \frac{60 \text{ kilometers}}{45 \text{ minutes}} \mid \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{60 \times 60 \text{ miles}}{45 \times 1.6 \text{ hour}} = 50 \text{ mph}$$

given value

(mph is an abbreviation for miles per hour)

Section 4.6, continued
Dimensional Analysis

Practice

Answer each of the following dimensional analysis problems. Show your work and write the solution in the space provided.

1. You are traveling on an interstate highway in the United States. The speed limit is 65 miles per hour. If you cross over into Canada where the speed limit is in kilometers per hour, what will the speed limit sign say? Keep in mind that 1 mile is approximately 1.6 kilometers.
-

2. Your car holds 18.2 gallons of gasoline. If one gallon is four quarts, and one quart is 1.06 liters, how many liters of gasoline would it take to fill your tank if it is completely empty.
-

4. You want to print an 8 inch \times 10 inch picture on your inkjet printer, but the page size options do not list 8 \times 10 as a page size. There are, however, several page sizes listed in millimeter dimensions. Since each page size has a unique set of dimensions, you only have to convert one dimension from inches to millimeters to identify the proper choice. If 1 inch is equal to 2.54 centimeters, and 1 centimeter is equal to 10 millimeters, what would 8 inches be in millimeters?
-

5. A hummingbird can fly at the rate of 88 feet per second. If one hour is equal to 3600 seconds and one mile is equal to 5,280 feet, how fast can a hummingbird fly in miles per hour?
-

6. Convert 20 miles per gallon into kilometers per liter. Consider that 1 gallon is approximately equal to 3.8 liters and 1 mile is approximately equal to 1.6 kilometers.
-

The Language of Algebra

Section 4 Review

Answer each question below. Darken the circle that represents the correct answer.

1. The herb, stevia, can be 200 times sweeter than refined sugar from sugar cane or sugar beets. If x represents the sweetness of sugar, which of the following could represent the sweetness of stevia?

A $200 + x$

B $x - 200$

C $\frac{x}{200}$

D $200x$

(A) (B) (C) (D)

4. Lacy purchased 10 packages of markers. Before tax was added, the total cost of the 10 packages was equal to the cost of 5 packages plus \$7.50. Which of the following equations could be used to determine p , the cost of 1 package?

A $10p = 5p + 7.5$

B $10 + 5 = 7.5p$

C $10p + 5p = 7.5$

D $\frac{p}{10} = 5p + 7.5$

(A) (B) (C) (D)

2. In a jump rope competition, Megan jumped 3 minutes less than twice as long as Pete. If P represents the length of time that Pete jumped, which of the following would represent the time that Megan jumped?

A $3 - P$

B $3 - 2P$

C $2P - 3$

D $2P + 3$

(A) (B) (C) (D)

5. The school bus used to take the cheerleaders to summer camp gets about 12 miles per gallon. If the camp is 300 miles away, which of the following is the CLOSEST number of gallons of gas they will need to get to camp?

A 2.5 gallons

B 25 gallons

C 340 gallons

D 3,400 gallons

(A) (B) (C) (D)

3. Andre purchased two tickets for the World Series. The price of the two tickets was three times more than the regular season ticket price of \$34 for one ticket. Which of the following equations could be used to determine c , the cost for one World Series ticket?

A $c = 2 \cdot 3 \cdot 34$

B $2c = 3 \cdot 34$

C $3c = 2 \cdot 34$

D $c + 3 = 34 - 2c$

(A) (B) (C) (D)

6. The Tanner family travels the interstate at 60 miles per hour. At that rate, how far can they travel in 4 hours?

A 15 miles

B 60 miles

C 64 miles

D 240 miles

(A) (B) (C) (D)

Section 4 Review, continued

7. A certain company has the following vacation plan. All employees automatically get 5 days of vacation. Employees also earn 2 days of vacation for each year they have worked at the company. Which equation represents V , the total days for an employee with x years of employment?

A $V = 5 + \frac{2}{x}$

B $V = 5 - \frac{2}{x}$

C $V = 2 + 5x$

D $V = 5 + 2x$

(A) (B) (C) (D)

10. The average starting salary for a chemical engineer (with 0 years experience) is \$46,700. For each year of experience, the engineer can earn an additional \$4,440. What is the equation that represents the yearly salary, Y , of a chemical engineer with N years experience?

A $Y = 4,440N + 46,700$

B $Y = -4,440N + 46,700$

C $Y = \frac{4,400}{N} + 46,700$

D $Y = (46,700 - 4,400)N$

(A) (B) (C) (D)

8. Brandon buys 7 bags of potato chips for a party. Some of the bags cost \$1.39 each. The others cost \$2.05 each. Which equation can be used to determine c , the total cost of all 7 bags of chips?

A $c = 1.39 + 2.05x$

B $c = 1.39x + 2.05(x + 7)$

C $c = 1.39x + 2.05(x - 7)$

D $c = 1.39x + 2.05(7 - x)$

(A) (B) (C) (D)

11. A gasoline company offers a credit card rewards program that gives points for every purchase made with the card. For every dollar spent on gasoline, the credit card user gets five reward points. For all other purchases, the user gets two points per dollar spent. If a user spends \$500 on the card and x dollars are for gas, which equation could represent the total points, p , earned by the user?

A $p = 2x + 5(500 - x)$

B $p = 5x + 2(500 - x)$

C $p = 5x + 2(500 + x)$

D $p = 2x + 5(500 + x)$

(A) (B) (C) (D)

9. Micah has \$50 in his savings account, and he adds \$10 each week. Which of the following could be used to represent the amount of money he has in savings after x weeks?

A $10x + 50$

B $10x - 50$

C $50x + 10$

D $50 - 10x$

(A) (B) (C) (D)

12. Roto-Plumber charges Mr. Nickels \$240 to repair his sink. The charge is based on a fixed fee of \$100 plus \$60 per hour of repair time. Which of the following could be used to find h , the number of repair hours?

A $240 = 100 + 60h$

B $240 = 100h + 60$

C $240h = 100 + 60h$

D $240h = 100h + 60$

(A) (B) (C) (D)

Section 4 Review, continued

13. Henry wants to buy a new pair of shoes that are on sale for \$49.95. If sales tax is 5%, how much will he pay for the shoes with the tax?

- A \$2.50
- B \$50.00
- C \$52.45
- D \$74.93

(A) (B) (C) (D)

16. An airplane averaged 540 miles per hour flying from coast to coast. How many miles per second is that?

- A 0.15 miles per second
- B 9 miles per second
- C 32,400 miles per second
- D 1,944,000 miles per second

(A) (B) (C) (D)

14. A real estate attorney makes a base salary of \$3,500 per month plus 3% of real estate sales. If she made \$5,225 last month, which equation could be used to find her real estate sales, s ?

- A $5,225 = 3,500 + 0.03s$
- B $5,225 = 3,500 - 0.03s$
- C $5,225 = \frac{3,500}{0.03s}$
- D $5,225 = 3,500 \times 0.03s$

(A) (B) (C) (D)

17. Janet walks 4 miles per hour and jogs 6 miles per hour. She walks and jogs for 3 hours. If she walks for x hours, which equation could be used to find M , the total miles that she walks and jogs?

- A $M = 4x + 3(6 - x)$
- B $M = 6x + 3(4 - x)$
- C $M = 4x + 6(3 - x)$
- D $M = 3 - (4 + 6)x$

(A) (B) (C) (D)

15. Kat's Korner Company makes cat scratching posts. The material costs for each scratching post is \$6.17. The company sells each post for \$57. The company's goal is to make \$10,000 profit per month on scratching post sales. Which of the following equations could be used to find the number of posts, p , the company needs to sell each month to make its goal?

- A $57 = 10,000 - 6.17p$
- B $10,000 = (57 - 6.17)p$
- C $10,000 = \frac{(57 - 6.17)}{p}$
- D $10,000 = \frac{p}{(57 - 6.17)}$

(A) (B) (C) (D)

18. Which of the following situations could be represented by the equation $A = B + 100$?

- A Karen's commission for last month, A , is \$100 less than her commission this month, B . Write an equation of last month's commission amount in terms of this month's.
- B *Brand A* potato chips have 100 more calories per serving than *Brand B*. How many calories are in one serving of *Brand A* in terms of the calories in *Brand B*?
- C Club membership fees are \$100 per member each year. Write an equation that shows total fees collected, A , for B number of members.
- D The speed of a race car was recorded as 100 miles per hour around a small track. Write an equation that would show how many hours it would take, A , to complete a B mile race.

(A) (B) (C) (D)