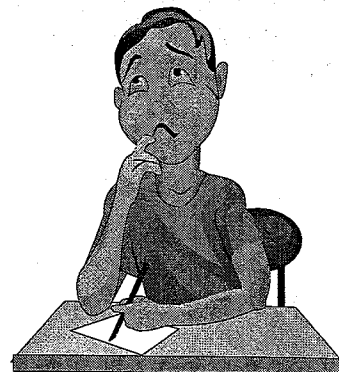


Exponents and Roots

Section 3.1 Multiplying and Dividing with Exponents



You saw exponents and roots briefly in the last section, but now let's review some of the rules for simplifying exponents and roots.

Multiplying with Exponents

When a number or variable is followed by an exponent, it is said to be "raised to a power." The exponent is the "power." When the bases of powers are the same, you can multiply them by adding the exponents and using the sum as the exponent with the common base. If you think about it, you can see why it works.

**Universal Formula for
Multiplying with Exponents**

$$a^m \cdot a^n = a^{m+n}$$

Example 1: Simplify the expression $a^3 \cdot a^5$.

$$a^3 \cdot a^5 = a^{3+5} = a^8$$

$$\underbrace{a \cdot a \cdot a} \cdot \underbrace{a \cdot a \cdot a \cdot a \cdot a} = a^8$$

When you count the a 's, you see that there are eight of them. That's the same as adding the exponents together.

Example 2: Simplify the expression $2^2 \cdot 2^3$.

$$2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$$

$$\underbrace{2 \cdot 2} \cdot \underbrace{2 \cdot 2 \cdot 2} = 2^5 = 32$$

Caution: Make sure the bases are the same. It doesn't work if they are not.

Dividing With Exponents

Dividing quantities with exponents is a similar process, but instead of adding the exponents, you subtract the exponent in the denominator from the exponent of the numerator. Keep in mind, the bases still have to be the same. The base can be any number except zero. (The " \neq " sign means "not equal to.")

**Universal Formula for
Dividing with Exponents**
 $a \neq 0$

$$\frac{a^m}{a^n} = a^{m-n}$$

Example 3: Simplify the expression $a^5 \div a^3$.

To understand why the universal formula for dividing exponents works, translate the division symbol into the vertical format with a fraction bar.

$$a^5 \div a^3 = \frac{a^5}{a^3} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a}$$

Next, cancel the common factors in the numerator and denominator. When you cancel the common factors, it leaves $a \cdot a$ or a^2 .

$$\frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a^2$$

As you can see, that's the same as subtracting the exponent in the denominator from the exponent in the numerator.

$$\frac{a^5}{a^3} = a^{5-3} = a^2$$

Section 3.1, continued Multiplying and Dividing with Exponents

Zero as an Exponent

Can you think of an instance when zero would be an exponent? What would the value be? One way to get a zero as an exponent would be to divide a power by itself. Check it out.

Universal Formula for Zero as a Power

$$a^0 = 1$$

You can divide two powers with the same base by subtracting the exponents. Pick any exponent and base (as long as the base is not zero). Let's use an exponent of 3 with a base of a . That would make a to the power of $3 - 3$, or a to the zero power. A base raised to any power divided by itself would have an exponent of zero — but what about the value?

$$\frac{a^3}{a^3} = a^{3-3} = a^0 = 1$$

What else can you say about a number divided by itself? Exactly — it's always one. So any number to the zero power would be one. **NOTE:** When simplifying problems with exponents, be sure that you never leave a base to the power of zero. Make it equal to 1.

Simplifying Exponential Expressions

Now, let's look at how you use these rules to simplify exponential expressions.

Example 4: Simplify the expression $2a^2 \cdot 3a^3$.

You use the commutative property of multiplication and the property for multiplying exponents to rewrite this expression and then simplify.

$$\begin{aligned} 2a^2 \cdot 3a^3 \\ 2 \cdot 3 \cdot a^{2+3} = 6a^5 \end{aligned}$$

Example 5: Simplify the expression $2a^5 \div 4a^2$.

Division is similar. The coefficients of 2 and 4 are simplified, and the variable, a , is also simplified.

$$\frac{2a^5}{4a^2} = \frac{\cancel{2}a^{5-2}}{\cancel{4}} = \frac{a^3}{2} \text{ or } \frac{1}{2}a^3$$

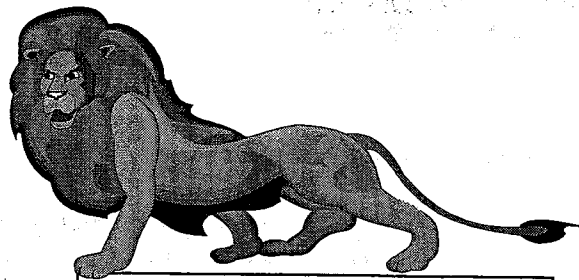
Practice

Use the rules for multiplying and dividing exponents to simplify each expression below.

_____ 1. $2^2 \cdot 2^3$	_____ 2. $4c^3 \cdot 4c^3$	_____ 3. $8p^4 \div 4p^3$
_____ 4. $2^3 \div 2^2$	_____ 5. $5b^7 \cdot 3b^2$	_____ 6. $3m^4 \div 3m^4$
_____ 7. $y^5 \cdot y^6$	_____ 8. $r^2 \cdot 2r^3$	_____ 9. $7x^3 \div 21x^2$

Exponents and Roots

Section 3.2 Powers Raised to Powers



Universal Power of a Power Property

$$(ab)^m = a^m \cdot b^m$$

Power of a Power

When an expression in parentheses is raised to a power, you can remove the parentheses by multiplying the exponents of each number or variable in the parentheses by the exponent outside the parentheses.

When there are multiple factors, the power outside the parentheses is applied to each.

Example 1: Simplify the expression $(a^3)^2$.

$$(a^3)^2 \rightarrow a^3 \cdot a^3 = a^{3 \cdot 2} = a^6$$

$$\underbrace{a \cdot a \cdot a} \cdot \underbrace{a \cdot a \cdot a} = a^6$$

Example 2: Simplify the expression $(3^2)^2$.

$$(3^2)^2 \rightarrow 3^{2 \cdot 2} = 3^4 = 81$$

Example 3: Simplify the expression $(3x^2)^2$.

When you have more than one factor in parentheses raised to a power outside the parentheses, you must distribute the power to each factor. Then do the math and use your exponent rules to simplify.

$$(3x^2)^2 \rightarrow (3)^2 \cdot (x^2)^2 = 9x^{2 \cdot 2} = 9x^4$$

Practice 1

Use the rule for raising a power to a power to simplify each expression below.

_____ 1. $(4^2)^2$	_____ 2. $(x^4)^3$	_____ 3. $(5y^2)^2$
_____ 4. $(2^2)^3$	_____ 5. $(v^3)^5$	_____ 6. $(2x)^3$

Power of a Quotient

When you have a rational number (i.e. a number written as a fraction) in parentheses raised to a power, the parentheses can be removed by applying the power to all the factors in the numerator and all the factors in the denominator.

Universal Power of a Quotient Property $b \neq 0$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Section 3.2, continued
Powers Raised to Powers

Example 4: Simplify the expression $\left(\frac{4}{5^2}\right)^2$.

In this example, the numerator is four to the first power. The denominator is five squared. To remove the parentheses, both the factors in the numerator and the denominator are squared.

$$\left(\frac{4}{5^2}\right)^2 \rightarrow \frac{4^2}{5^{2 \cdot 2}} = \frac{4^2}{5^4} = \frac{16}{625}$$

Example 5: Simplify the expression $\left(\frac{3x^3}{4}\right)^3$.

This problem might look more difficult, but just take it one step at a time. Each factor inside the parentheses needs to be cubed. In the numerator, you must cube the 3 and the x^3 . In the denominator, you must cube the 4. Use the power of a power rule for the $(x^3)^3$, and then simplify.

$$\left(\frac{3x^3}{4}\right)^3 \rightarrow \frac{3^3 \cdot x^{3 \cdot 3}}{4^3} = \frac{27x^9}{64}$$

Practice 2

Use the rule for raising a quotient to a power to simplify each expression below.

_____ 1. $\left(\frac{2^2}{3}\right)^2$	_____ 2. $\left(\frac{x^3}{2}\right)^2$	_____ 3. $\left(\frac{2a^2}{5}\right)^2$
_____ 4. $\left(\frac{1}{2^3}\right)^2$	_____ 5. $\left(\frac{3}{y^3}\right)^3$	_____ 6. $\left(\frac{4b^2}{7}\right)^2$

Mixed Practice 3.1 – 3.2

Use rules of exponents to simplify the following expressions.

_____ 1. $x^2 \cdot x^4$	_____ 2. $(a^4)^5$	_____ 3. $\left(\frac{2}{9}\right)^2$
_____ 4. $2b^5 \cdot 3b^3$	_____ 5. $(3y^2)^3$	_____ 6. $\left(\frac{b^3}{4}\right)^2$
_____ 7. $9p^6 \div 3p^2$	_____ 8. $(2b^3)^4$	_____ 9. $\left(\frac{2p^4}{3}\right)^3$

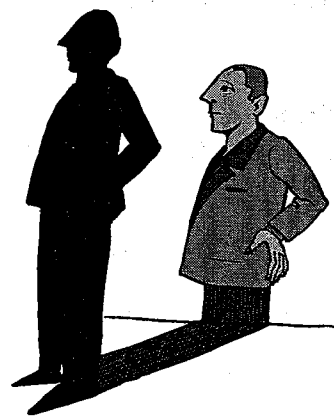
Exponents and Roots

Section 3.3 Negative Exponents

One way to think of negative exponents is to rewrite the power as a rational number (fraction) with one as the numerator and the power as the denominator. Or you could say that positive exponents go in the numerator and negative exponents move to the denominator and change their sign to positive. (A negative exponent in the denominator would move to the numerator.)

Rule for Negative Exponents

$$a^{-m} = \frac{1}{a^m}$$



Example 1: Simplify the expression $(3)^{-3}$.

Using the rule for negative exponents, the power is moved to the denominator. Then, the exponent can be simplified.

$$3^{-3} \rightarrow \frac{1}{3^3} = \frac{1}{27}$$

If you have a multiplication problem to simplify, move factors with negative exponents into the denominator and keep factors with positive exponents in the numerator. Then you can simplify. Example 2 below shows how.

Example 2: Simplify the expression $3^{-2} \cdot 6^2$.

Keep the 6^2 in the numerator since it has a positive exponent, but move the 3^{-2} to the denominator and change the negative exponent to a positive one. Then do the math.

$$6^2 \cdot 3^{-2} \rightarrow 6^2 \cdot \frac{1}{3^2} = \frac{6^2}{3^2} = \frac{36}{9} = 4$$

shortcut

Now let's look at a couple of division problems. When a division problem is written with a “ \div ” sign, you can rewrite it as a fraction in two different ways. Choose the way that makes the most sense to you.

Example 3: Simplify the expression $a^{-4} \div a^{-3}$.

Step 1: First, write each factor in fraction form.

$$a^{-4} \div a^{-3} \rightarrow \frac{1}{a^4} \div \frac{1}{a^3}$$

Step 2: Remember, dividing by a fraction is the same as multiplying by its inverse.

$$\frac{1}{a^4} \cdot \frac{a^3}{1} = \frac{a^3}{a^4} = \frac{1}{a}$$

Step 3: Simplify by using the rules of exponents.

Example 4: Simplify the expression $2a^{-3} \div 3a^{-5}$.

Step 1: The “ \div ” sign can be replaced with a fraction bar. Rewrite as a fraction before changing the negative exponents. You may see problems written as fractions this way instead of with a “ \div ” sign.

$$2a^{-3} \div 3a^{-5} \rightarrow \frac{2a^{-3}}{3a^{-5}}$$

Step 2: For any variable that has a negative exponent, move it to the other side of the fraction bar and change the exponent to a positive. Be **careful: only move the variables and not the coefficients!**

$$\frac{2\boxed{a^{-3}}}{3\boxed{a^{-5}}} \rightarrow \frac{2a^5}{3a^3}$$

Step 3: Now simplify using the rules of exponents.

$$\frac{2a^2}{3} \text{ or } \frac{2}{3}a^2$$

Section 3.3, continued
Negative Exponents

Practice 1

Use the rule for negative exponents to simplify each expression below.

_____ 1. $(4)^{-2}$	_____ 2. $a^{-5} \cdot a^3$	_____ 3. $5y^2 \div 2y^{-3}$
_____ 4. x^{-4}	_____ 5. $y^{-2} \cdot y^{-4}$	_____ 6. $\frac{2x^{-1}}{3x^{-3}}$
_____ 7. $(2)^{-2} \cdot (3)^2$	_____ 8. $2x^2 \cdot 5x^{-1}$	_____ 9. $\frac{b}{2b^2}$
_____ 10. $(2x)^{-2}$	_____ 11. $4b^3 \cdot b^{-3}$	_____ 12. $m^{-2} \div m^{-2}$

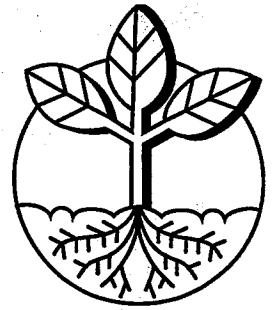
Mixed Practice 3.1 – 3.3

Use rules of exponents to simplify the following expressions.

_____ 1. $x^2 \cdot 2x^3$	_____ 2. $3b^3 \cdot 2b$	_____ 3. $4y^3 \cdot 2y^4$
_____ 4. $3x^2 \div x$	_____ 5. $6z^6 \div 3z^3$	_____ 6. $2x^5 \div 4x^3$
_____ 7. $(2^3)^2$	_____ 8. $(c^4)^3$	_____ 9. $(3z^2)^2$
_____ 10. $\left(\frac{2}{5}\right)^2$	_____ 11. $\left(\frac{x^3}{3}\right)^3$	_____ 12. $\left(\frac{1}{2y^3}\right)^3$
_____ 13. $2^3 \cdot 3^{-2}$	_____ 14. $3x^{-4}$	_____ 15. $\frac{2x^3}{3x^{-1}}$

Exponents and Roots

Section 3.4 Roots



In Section 1.1, you were introduced to roots. Now let's see how you simplify them.

Simplifying Roots by Factoring

Remember, roots and exponents are opposite operations. Let's see how they are related. Squaring a number gives you a product. To reverse the operation, you take the square root. (Remember, a square root doesn't usually give the "2" with the radical. It's understood to be a square root if no number is given.) Cubing a number gives you a product. To reverse the operation, you take the cube root. Are you beginning to see that in order to simplify a root, you should find the factors of the radicand? Let's look at an example of how that works.

$$5^2 = 25 \qquad \sqrt{25} = 5$$

$$2^3 = 8 \qquad \sqrt[3]{8} = 2$$

Example 1: Simplify the expression $\sqrt[4]{16}$.

The fourth root of 16 is like asking, "What number when taken to the fourth power equals to 16?" The factors of 16 are four 2's, so to answer the question, two to the fourth power equals 16. The answer is 2.

$$\sqrt[4]{16} \longrightarrow \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = 2$$

According to the rules of multiplication, $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. This rule is useful when simplifying roots that are not perfect multiples of a factor. The power of the root must remain the same. Look at the example below.

Multiplication Property of Roots

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Example 2: Simplify the expression $\sqrt{12}$.

Twelve can be factored into 4 and 3. The square root of 4 is 2, but the square root of 3 cannot be factored further. Therefore, the simplified answer is $2\sqrt{3}$.

$$\sqrt{12} \rightarrow \sqrt{4 \cdot 3}$$

$$\sqrt{4} \cdot \sqrt{3} = \sqrt{2 \cdot 2} \cdot \sqrt{3} = 2\sqrt{3}$$

Example 3: Simplify the expression $\sqrt[3]{16}$.

The cube root means you must have three of the same factor. You have three two's with one two left over.

$$\sqrt[3]{16} \longrightarrow \sqrt[3]{\underbrace{2 \cdot 2 \cdot 2}_{2^3} \cdot 2} = 2\sqrt[3]{2}$$

Practice

Simplify each expression below by using factoring.

_____ 1. $\sqrt[3]{27}$	_____ 2. $\sqrt{64}$	_____ 3. $\sqrt{200}$
_____ 4. $\sqrt{18}$	_____ 5. $\sqrt[3]{81}$	_____ 6. $\sqrt[3]{64}$
_____ 7. $\sqrt[4]{48}$	_____ 8. $\sqrt{72}$	_____ 9. $\sqrt[3]{40}$

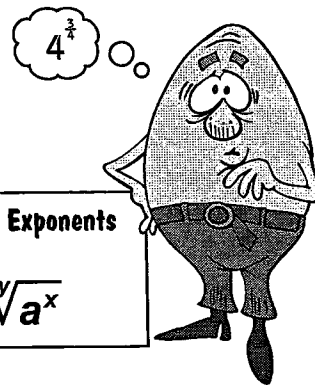
Exponents and Roots

Section 3.5 Fractional Exponents

Exponents can also be rational numbers (or fractions). The rule for fractional exponents shows how an expression with a fractional exponent can be converted to an expression with whole number exponents and roots. Let's look at two examples.

Rule for Fractional Exponents

$$a^{\frac{x}{y}} = \sqrt[y]{a^x}$$



Example 1: Simplify the expression $16^{\frac{1}{2}}$.

You can rewrite this fractional exponent as the square root of 16 raised to the first power. The numerator is the exponent, and the denominator is the root. Now you simply have the square root of 16, which is 4.

$$16^{\frac{1}{2}} \longrightarrow \sqrt[2]{(16)^1} = \sqrt{16} = 4$$

So, a number or expression raised to the $\frac{1}{2}$ power is the same as taking the square root of the number or expression. Now, check this one out. It's a little more complicated.

Example 2: Simplify the expression $3t^{\frac{2}{3}}$ when $t = 8$.

Step 1: First, you must substitute 8 for t .

$$3t^{\frac{2}{3}} \longrightarrow 3 \cdot 8^{\frac{2}{3}}$$

Step 2: Next, you convert the fractional exponent into whole number roots and exponents.

$$3\sqrt[3]{8^2}$$

Step 3: Use the multiplication property of roots to split (rewrite) the expression as the product of two roots.

$$3 \cdot \sqrt[3]{8} \cdot \sqrt[3]{8}$$

Step 4: In case you don't remember that the cube root of 8 is 2, you can factor each root and see that you get three two's.

$$3\sqrt[3]{2 \cdot 2 \cdot 2} \cdot \sqrt[3]{2 \cdot 2 \cdot 2}$$

Step 5: You take the cube root of each 8 and get two for both.

$$3 \cdot 2 \cdot 2$$

Step 6: The final step is to do the multiplication to get 12.

$$12$$

Practice

Simplify each expression below by using the rule for fractional exponents and factoring.

_____ 1. $25^{\frac{1}{2}}$	_____ 2. $8^{\frac{2}{3}}$	_____ 3. $12^{\frac{1}{2}}$
_____ 4. $27^{\frac{1}{3}}$	_____ 5. $16^{\frac{3}{4}}$	_____ 6. $32^{\frac{2}{5}}$
_____ 7. $3x^{\frac{1}{3}}$ when $x = 125$	_____ 8. $2t^{\frac{2}{3}}$ when $t = 64$	_____ 9. $3y^{\frac{3}{4}}$ when $y = 81$

Exponents and Roots

Section 3 Review

Answer each question below. Darken the circle that represents the correct answer.

1. Which of the following is equivalent to $3x^2 \cdot 2x$?

- A $5x^2$
- B $5x^3$
- C $6x^2$
- D $6x^3$

(A) (B) (C) (D)

5. Which of the following is equivalent to 4^{-1} ?

- A $\frac{1}{4}$
- B $-\frac{1}{4}$
- C 3
- D -4

(A) (B) (C) (D)

2. Which of the following is equivalent to $6y^6 \div 4y^2$?

- A $\frac{3}{2}y^3$
- B $\frac{3}{2}y^4$
- C $2y^3$
- D $2y^4$

(A) (B) (C) (D)

6. Which of the following is equivalent to $2^{-3} \cdot 2^2$?

- A $\frac{1}{2}$
- B -2
- C -24
- D -32

(A) (B) (C) (D)

3. Which of the following is equivalent to $(4t^2)^3$?

- A $4t^5$
- B $4t^6$
- C $64t^5$
- D $64t^6$

(A) (B) (C) (D)

7. Which of the following is equivalent to $2x^6 \cdot x^{-2}$?

- A $2x^{-12}$
- B $2x^{-3}$
- C $2x^4$
- D x^6

(A) (B) (C) (D)

4. Which of the following is equivalent to $\left(\frac{3^2}{2}\right)^3$?

- A $\frac{9}{2}$
- B $\frac{243}{8}$
- C $\frac{729}{8}$
- D 3

(A) (B) (C) (D)

8. Which of the following is equivalent to $\left(\frac{4a^3}{3}\right)^2$?

- A $\frac{4a^5}{3}$
- B $\frac{16a^6}{3}$
- C $\frac{16a^5}{9}$
- D $\frac{16a^6}{9}$

(A) (B) (C) (D)

Section 3 Review, continued

<p>9. Which of the following is equivalent to $y^3 \div y^{-2}$?</p> <p>A y^{-6}</p> <p>B y</p> <p>C y^5</p> <p>D $\frac{1}{y}$</p> <p>(A) (B) (C) (D)</p>	<p>13. Which of the following is equivalent to $\sqrt[4]{625}$?</p> <p>A 5</p> <p>B 25</p> <p>C 125</p> <p>D undefined</p> <p>(A) (B) (C) (D)</p>
<p>10. Which of the following is equivalent to $\frac{2x^{-3}}{4x^{-1}}$?</p> <p>A $\frac{2}{x^2}$</p> <p>B $\frac{1}{2x^2}$</p> <p>C $\frac{x^2}{2}$</p> <p>D $\frac{8}{x^4}$</p> <p>(A) (B) (C) (D)</p>	<p>14. Which of the following is equivalent to $81^{\frac{3}{4}}$?</p> <p>A 3</p> <p>B 9</p> <p>C 27</p> <p>D 60.75</p> <p>(A) (B) (C) (D)</p>
<p>11. Which of the following is equivalent to $5x^5 \cdot 3x^{-3}$?</p> <p>A $15x^2$</p> <p>B $\frac{5}{3x^2}$</p> <p>C $\frac{15}{x^2}$</p> <p>D $\frac{1}{15x^2}$</p> <p>(A) (B) (C) (D)</p>	<p>15. Which of the following is equivalent to $3 \cdot 16^x$ when $x = \frac{1}{2}$?</p> <p>A 4</p> <p>B 12</p> <p>C 16</p> <p>D 24</p> <p>(A) (B) (C) (D)</p>
<p>12. Which of the following is equivalent to $\sqrt{50}$?</p> <p>A $2\sqrt{5}$</p> <p>B $5\sqrt{2}$</p> <p>C $5\sqrt{10}$</p> <p>D 25</p> <p>(A) (B) (C) (D)</p>	<p>16. Which of the following is equivalent to $4p^{\frac{2}{3}}$ when $p = 8$?</p> <p>A 4</p> <p>B 8</p> <p>C 12</p> <p>D 16</p> <p>(A) (B) (C) (D)</p>