DeSoto
COUNTY SCHOOLS

Geometry

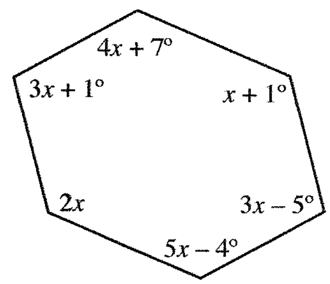
Week 7

POLYGONS 8.2.1 – 8.2.2

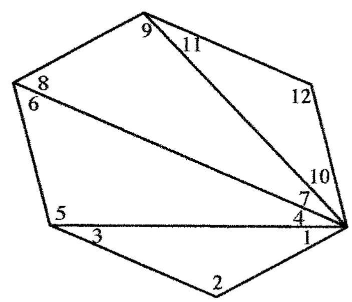
After studying triangles and quadrilaterals, students now extend their study to all polygons, with particular attention to regular polygons, which are equilateral and equiangular. Using the fact that the sum of the measures of the angles in a triangle is $180°$, students describe a method to determine the sum of the measures of the interior angles of any polygon. Next they explore the sum of the measures of the exterior angles of a polygon. Finally they use the information about the angles of polygons along with their triangle tools to determine the angle measures and areas of regular polygons.

For additional information see the Math Notes boxes in Lessons 8.2.2 and 8.4.1.

Example 1



The figure at right is a hexagon. What is the sum of the measures of the interior angles of a hexagon? Explain how you know. Then write an equation and solve for x.



One way to calculate the sum of the interior angles of the hexagon is to divide the polygon into triangles. One way to divide the hexagon into triangles is to draw all of the diagonals from a single vertex, as shown at right. Doing this forms four triangles, each with angle measures summing to $180°$.

$$\underbrace{m\angle1+m\angle2+m\angle3}_{180°}+\underbrace{m\angle4+m\angle5+m\angle6}_{180°}+\underbrace{m\angle7+m\angle8+m\angle9}_{180°}+\underbrace{m\angle10+m\angle11+m\angle12}_{180°}$$
$$=4(180°)=720°$$

Note: Students may notice that the number of triangles drawn from a single vertex is always two less than the number of sides. This example illustrates why the sum of the interior angles of a polygon may be calculated using the formula sum of interior angles = $180°(n-2)$, where n is the number of sides of the polygon.

Now using the sum of the angles, write an equation, and solve for x.

$$(3x + 1°) + (4x + 7°) + (x + 1°) + (3x - 5°) + (5x - 4°) + (2x) = 720°$$

$$18x = 720°$$

$$x = 40°$$

Example 2

If the sum of the measures of the interior angles of a polygon is 2340°, how many sides does the polygon have?

Use the formula sum of interior angles = $180°(n-2)$ to write an equation and solve for n. The solution is shown at right.

$$\begin{aligned} 180°(n-2) &= 2340° \\ 180°n - 360° &= 2340° \\ 180°n &= 2700° \\ n &= 15 \end{aligned}$$

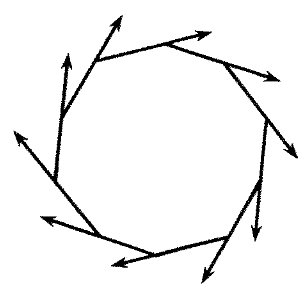
Since $n = 15$, the polygon has 15 sides.

It is important to note that if the answer is not a whole number, then either an error was made or there is no polygon with interior angles that sum to the given measure. Since the answer is the number of sides, the answer must be a whole number. Polygons cannot have "7.2" sides!

Example 3

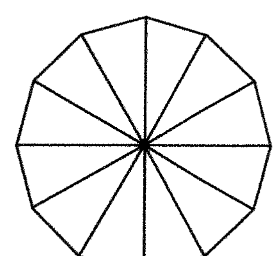
What is the measure of an exterior angle of a regular decagon?

A decagon is a 10-sided polygon. The sum of the measures of the exterior angles of any polygon, one at each vertex, is always 360°, no matter how many sides the polygon has. In this case the ten exterior angles are congruent since the decagon is regular. Therefore, each angle measures $\frac{360°}{10} = 36°$.



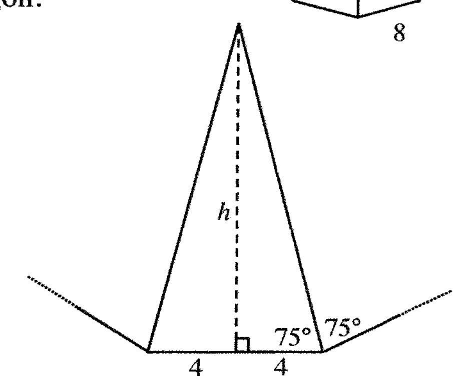
Example 4

A regular dodecagon (12-sided polygon) has a side length of 8 cm. What is the area of the dodecagon?



Imagine dividing the dodecagon into twelve congruent triangles, radiating from the center, as shown at right. If we determine the area of one of the triangles, then we can multiply it by twelve to get the area of the entire dodecagon.

One of the triangles is enlarged at right. The triangle is isosceles, so drawing a segment from the vertex angle perpendicular to the base is the height. Because the triangle is isosceles, the height bisects the base.



Calculate the sum of all the interior angles of the dodecagon by using the formula $(180°)(12-2) = 1800°$.

Since it is a regular dodecagon, all the interior angles are congruent, so each angle measures $1800° \div 12 = 150°$. The segments radiating from the center form congruent isosceles triangles, so the base angles of each triangle measure 75° (half of the 150° angle, as shown in the diagram on the previous page). Use trigonometry to calculate the value of h as shown as right. It is best to use an unrounded value of h to calculate the area, and then round the answer appropriately at the end of your calculations.

$$\tan 75° = \frac{h}{4}$$
$$h = 4 \tan 75°$$
$$h \approx 14.928 \text{ cm}$$

Therefore the area of one of these triangles is: $A \approx \frac{1}{2}(8 \text{ cm})(14.928 \text{ cm}) \approx 59.713 \text{ cm}^2$

Multiply the area of one triangle by 12 to get the area of the entire dodecagon.

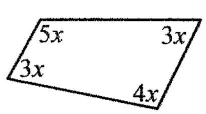
$$A \approx 12(59.713 \text{ cm}) \approx 716.55, \text{ or about } 717 \text{ cm}^2$$

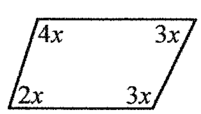
Problems

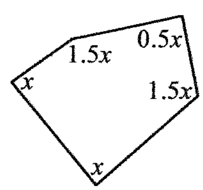
Calculate the measures of the angles in each problem below.

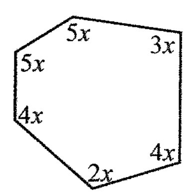
1. The sum of the interior angles of a heptagon (7-gon).
2. The sum of the interior angles of an octagon (8-gon).
3. The measure of each interior angle of a regular dodecagon (12-gon).
4. The measure of each interior angle of a regular 15-gon.
5. The measure of each exterior angle of a regular 17-gon.
6. The measure of each exterior angle of a regular 21-gon.

Solve for x in each of the figures below.

7. 

8. 

9. 

10. 

Complete each of the following problems.

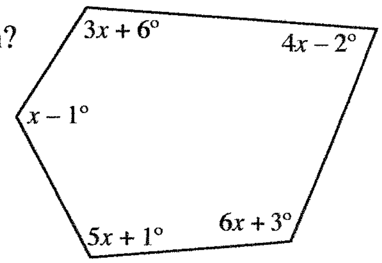
11. Each exterior angle of a regular n-gon measures $16\frac{4}{11}°$. How many sides does this n-gon have?
12. Each exterior angle of a regular n-gon measures $13\frac{1}{3}°$. How many sides does this n-gon have?
13. Each interior angle of a regular n-gon measures 156°. How many sides does this n-gon have?
14. Each interior angle of a regular n-gon measures 165.6°. How many sides does this n-gon have?

15. What is the area of a regular pentagon with side length 8.0 cm?

16. Calculate the area of a regular hexagon with side length 10.0 ft.

17. Calculate the area of a regular octagon with side length 12.0 m.

18. What is the area of a regular decagon with side length 14.0 in?



19. Using the pentagon at right, write an equation and solve for x.

20. What is the sum of the measures of the interior angles of a 14-sided polygon?

21. What is the measure of each interior angle of a regular 16-sided polygon?

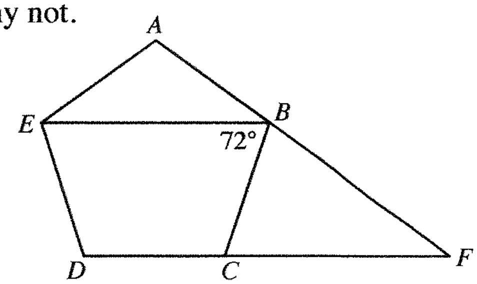
22. What is the sum of the measures of the exterior angles of a decagon (10-gon)?

23. Each exterior angle of a regular polygon measures 22.5°. How many sides does the polygon have?

24. Is there a polygon with interior angle measures that add up to 3060°? If so, how many sides does it have? If not, explain why not.

25. Is there a polygon with interior angle measures that add up to 1350°? If so, how many sides does it have? If not, explain why not.

26. Is there a polygon with interior angle measures that add up to 4410°? If so, how many sides does it have? If not, explain why not.



27. In the figure at right, *ABCDE* is a regular pentagon. Is $\overline{EB} \parallel \overline{DF}$? Justify your answer.

28. What is the area of a regular pentagon with a side length of 10 units?

29. What is the area of a regular 15-gon with a side length of 5 units?

Worksheet #1 Chapter 8

Name ______________________

No rounding. All numerical answers must be proper fractions, integers, mixed numbers, terminating decimals, or simplified radicals.

The number of sides of a convex polygon is given. Find the sum of the measures of the interior angles of each polygon.

1) 8 2) 12 3) 14 4) 16 5) p

The sum of the measures of the interior angles of a convex polygon is given. Find the number of sides of each polygon.

6) 7020° 7) 1980° 8) 6120° 9) 1800° 10) 3420°

The number of sides of a regular polygon is given. Find the measure of each interior angle of each polygon.

11) 7 12) 9 13) 11 14) 15 15) 17

Find the exact measure of each exterior angle of the regular polygon.

16) pentagon 17) heptagon 18) decagon 19) 18-gon 20) 20-gon

21) Home plate on a baseball field has three right angles and two congruent angles. Find the measure of the two congruent angles.

22) The sum of the measures of seven angles of an octagon is 1000°. Find the measure of the eighth angle.

23) How many sides does a regular polygon have if each exterior angle has a measure of 15°?

24) How many sides does a regular polygon have if each interior angle has a measure of 108°?

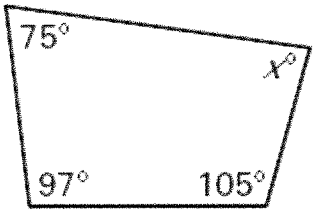
25) Find the number of sides of a polygon if the sum of the measures of its interior angles is twice the sum of the measures of its exterior angle.

26) The measure of each interior angle of a regular polygon is eight times that of an exterior angle. How many sides does the polygon have?

27) In quadrilateral ABCD the measures of, $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are the ratio of 1:2:3:4, respectively. Find the measures of the four angles.

Find the value of x.

28)



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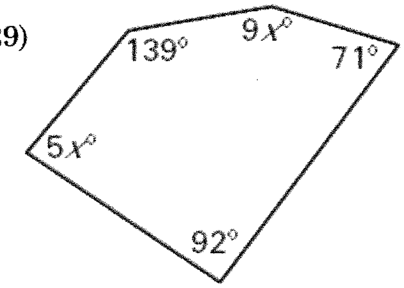

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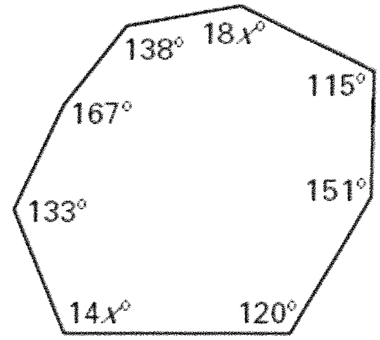

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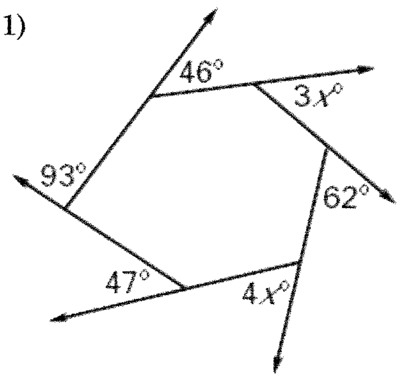

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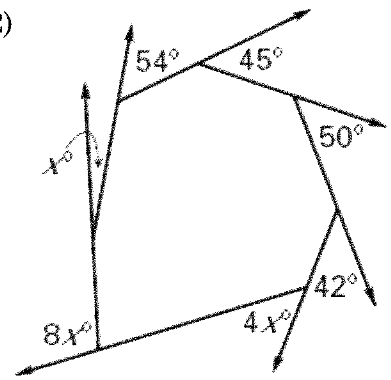

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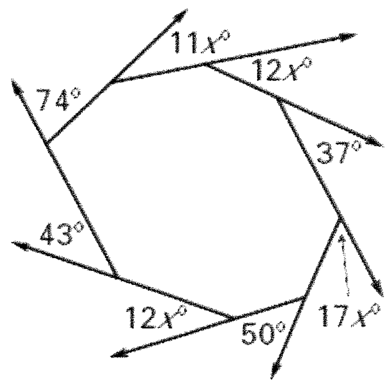
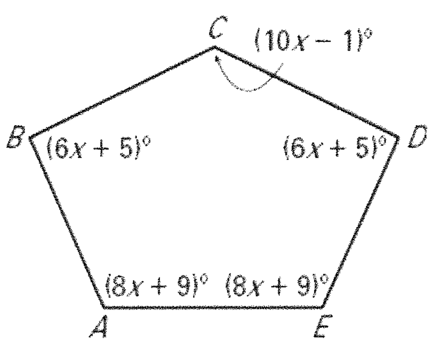
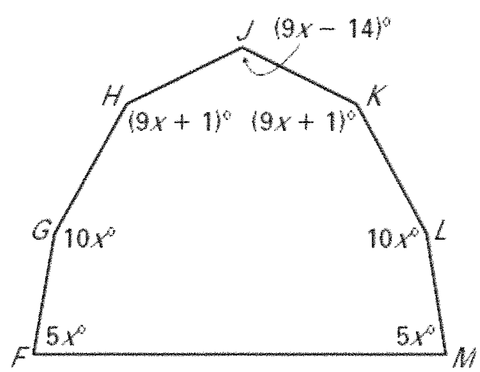


34) **Light Fixture** The side view of a light fixture is shown below. Find the value of x. Then determine the measure of each angle.



35) **Tent** The front view of a camping tent is shown below. Find the value of x. Then determine the measure of each angle.

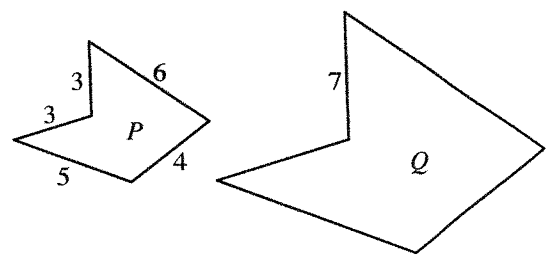


AREA RATIOS OF SIMILAR FIGURES 8.3.1 – 8.3.2

In this section, students return to similarity to explore what happens to the area of a figure if it is reduced or enlarged. In Chapter 2, students learned about the ratio of similarity, also called the "scale factor." If two similar figures have a ratio of similarity of $\frac{a}{b}$, then the ratio of their perimeters is also $\frac{a}{b}$, while the ratio of their areas is $\frac{a^2}{b^2}$.

Example 1

The polygons P and Q at right are similar.



a. What is the ratio of similarity?

b. What is the perimeter of polygon P?

c. Use your previous two answers to determine the perimeter of polygon Q.

d. If the area of polygon P is 20 square units, what is the area of polygon Q?

The ratio of similarity is the ratio of the lengths of two corresponding sides. In this case, use the side of P that corresponds to the side of Q that is labeled with its length. The ratio of similarity is $\frac{3}{7}$.

To calculate the perimeter of P, add all the side lengths: $3+6+4+5+3=21$. If the ratio of similarity of the two polygons is $\frac{3}{7}$ then the ratio of their perimeters is also $\frac{3}{7}$.

$$\frac{\text{perimeter } P}{\text{perimeter } Q}=\frac{3}{7}$$
$$\frac{21}{Q}=\frac{3}{7}$$
$$3Q=147$$
$$\text{perimeter } Q=49 \text{ units}$$

If the ratio of similarity is $\frac{3}{7}$ then the ratio of the areas is $\left(\frac{3}{7}\right)^2=\frac{9}{49}$.

$$\frac{\text{area } P}{\text{area } Q}=\left(\frac{3}{7}\right)^2$$
$$\frac{20}{Q}=\frac{9}{49}$$
$$9Q=980$$
$$\text{area } Q \approx 108.89 \text{ square units}$$

Example 2

Two rectangles are similar. If the area of one rectangle is 49 square units, and the area of the other rectangle is 256 square units, what is the ratio of similarity between these two rectangles?

Since the rectangles are similar, the ratio of their areas is $\frac{a^2}{b^2}$ and the the ratio of similarity is $\frac{a}{b}$. Using the given areas, the ratio of their areas is $\frac{49}{256}$. Therefore we can write:

$$\frac{a^2}{b^2} = \frac{49}{256}$$

$$\frac{a}{b} = \sqrt{\frac{49}{256}} = \frac{\sqrt{49}}{\sqrt{256}} = \frac{7}{16}$$

The ratio of similarity of the two rectangles is $\frac{7}{16}$.

Problems

1. If figure A and figure B are similar with a ratio of similarity of $\frac{5}{4}$, and the perimeter of figure A is 18 units, what is the perimeter of figure B?

2. If figure A and figure B are similar with a ratio of similarity of $\frac{1}{8}$, and the area of figure A is 13 square units, what is the area of figure B?

3. If figure A and figure B are similar with a ratio of similarity of 6, that is, 6 to 1, and the perimeter of figure A is 54 units, what is the perimeter of figure B?

4. If figure A and figure B are similar and the ratio of their perimeters is $\frac{17}{6}$, what is their ratio of similarity?

5. If figure A and figure B are similar and the ratio of their areas is $\frac{32}{9}$, what is their ratio of similarity?

6. If figure A and figure B are similar and the ratio of their perimeters is $\frac{23}{11}$, does that mean the perimeter of figure A is 23 units and the perimeter of figure B is 11 units? Explain.

Answers

1. 14.4 units 2. 832 sq. units 3. 9 units 4. $\frac{17}{6}$ 5. $\frac{\sqrt{32}}{\sqrt{9}} = \frac{4\sqrt{2}}{3} \approx \frac{5.66}{3}$

6. No, it just tells us the ratio. Figure A could have a perimeter of 46 units and figure B a perimeter of 22 units.

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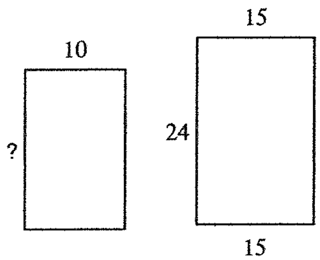
Using Similar Polygons

Date________________ Period____

The polygons in each pair are similar. Find the missing side length.

1)



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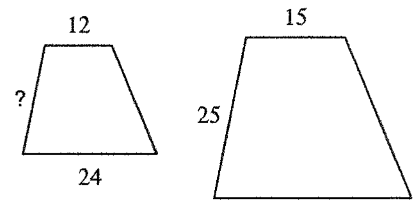

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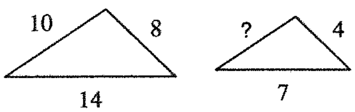

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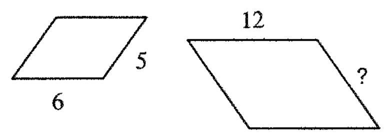

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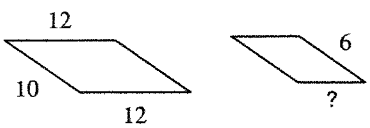

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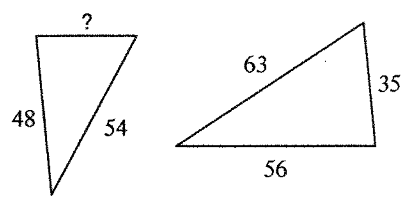

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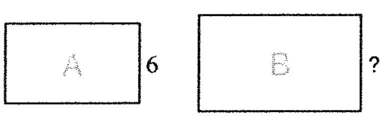
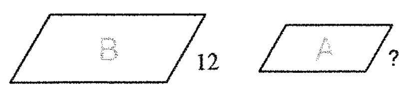


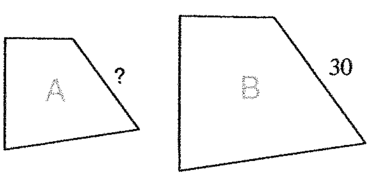
scale factor from A to B = 2 : 7

8)



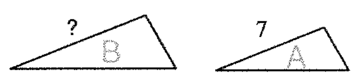
scale factor from A to B = 2 : 3

9)



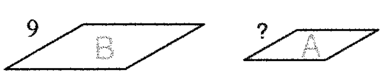
scale factor from A to B = 5 : 6

10)



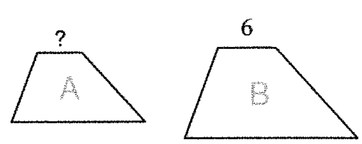
scale factor from A to B = 1 : 7

11)



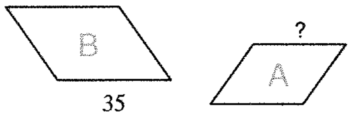
scale factor from A to B = 2 : 3

12)



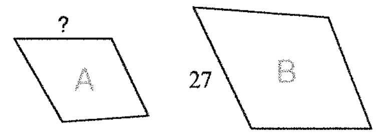
scale factor from A to B = 1 : 2

13)



scale factor from A to B = 6 : 7

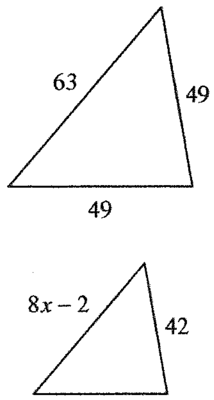
14)



scale factor from A to B = 1 : 3

Solve for x. The polygons in each pair are similar.

15)



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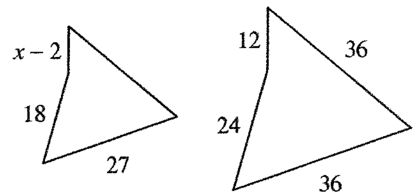

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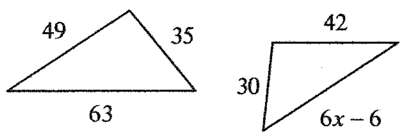

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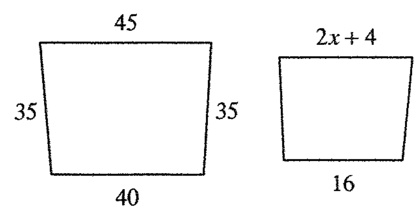

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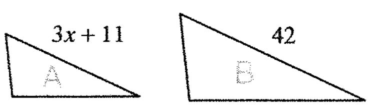
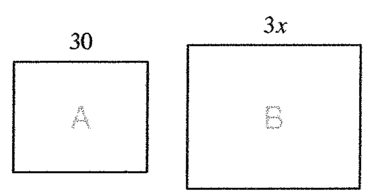


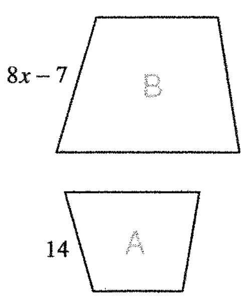
scale factor from A to B = 5 : 6

20)



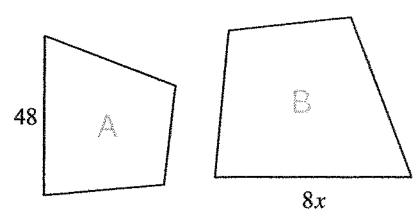
scale factor from A to B = 5 : 6

21)



scale factor from A to B = 2 : 7

22)



scale factor from A to B = 6 : 7