

Geometric Sequences and Series

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Geometric Sequence:

A sequence in which each term is a product of the preceding term and a common ratio called r .

$$8, 4, 2, 1, \dots$$

$$r = 4/8 = \frac{1}{2}$$

The n th term for a geometric sequence is represented by:

$$a_n = a_1 \cdot r^{n-1}$$

Example 1: Given the sequence 3, 12, 48, ...

$$r = \frac{12}{3} = 4$$

a) Find the 5th term

$$3, 12, 48, \underline{192}, \boxed{\underline{768}}$$

b) Find the 18th term

$$a_{18} = 3 \cdot 4^{18-1}$$

$$a_{18} = 51539667550$$

Example 2: Given the sequence 5, 15, 45, ...

$$r = \frac{15}{5} = 3$$

a) Find the 5th term

$$5, 15, 45, \underline{135}, \underline{405}$$

b) Find the 19th term

$$a_{19} = 5 \cdot 3^{19-1}$$

$$= 1,937,102,445$$

Example 3: Given the sequence 1000, 250, 62.5, ...

$$r = \frac{250}{1000} = \frac{1}{4}$$

Find the 8th term

$$\begin{aligned} a_8 &= 1000 \cdot \left(\frac{1}{4}\right)^{8-1} \\ &= \frac{125}{2048} \end{aligned}$$

Example 4: If the sixth term of a sequence is 192 and the ratio is 4, determine:

a) The 1st term

$$\frac{192}{4^{6-1}} = \frac{a_1 \cdot 4^{6-1}}{4^{6-1}}$$
$$a_1 = \frac{3}{16}$$

b) The 17th term

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Example 5: If the 3rd term of a sequence is 75 and the ratio is 3, determine:

a) The 1st term

$$75 = a_1 \cdot 3^{3-1}$$

$$\frac{75}{9} = \frac{a_1 \cdot 9}{9}$$

$$a_1 = 8\frac{1}{3}$$

b) The 20th term

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We can also determine missing terms by using a geometric mean(s). There are 3 in this problem since there are 3 blanks.

In other words, Divide the right number by the left number, and raise it to the $1/(\text{number of steps})$ exponent.

Example 6: Find the missing terms given

4, 12, 36, 108, 324

$$r = \left(\frac{324}{4} \right)^{1/4}$$
$$r = 3$$

Example 7: Find the missing terms given

48, -120, 300, -750

$$r = \left(\frac{-750}{48} \right)^{1/3}$$

$$r = -2.5$$

A geometric series is the sum of a geometric sequence.

It is represented by the following:

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

Example 8: Determine the sum of the first ten terms given $16 - 48 + 144 - 432 + \dots$

$$r = \frac{-48}{16} = -3$$

$$S_{10} = \frac{16 - 16(-3)^{10}}{1 - (-3)}$$

$$S_{10} = -236192$$

Example 9: Determine the sum of the first twenty terms given $6 + 12 + 24 + 48 + \dots$

$$S_{20} = \frac{6 - 6(2)^{20}}{1 - 2}$$

$$S_{20} = 6291450$$

Example 10: Add the first eight terms given

$$.04 + \underline{\quad} + 10 + \dots$$

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Homework

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